EIGENSPACE TECHNIQUES FOR ACTIVE FLUTTER SUPPRESSION
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Abstract

Eigenspace (ES) techniques are used to design an active flutter suppression system for the DAST ARW-2 flight test vehicle. The ES controller meets control surface activity specifications and at the flutter test condition provides reduced wing root torsion at the gust test condition, and results in improved flutter boundaries. The ES controller is compared with a controller designed using Linear Quadratic (LQ) techniques. The LQ controller exhibits better phase margins at the flutter condition than does the ES controller but the LQ design requires large feedback gains on actuator states while the ES does not. This results in reduced overall actuator gain for the LQ design.

Nomenclature

Vectors

u = control input

v_i = attainable closed loop eigenvector associated with λ_i eigenvalue

v_{d_i} = desired closed loop eigenvector associated with λ_i eigenvalue

w_i = vector used in calculation of ES gain matrix, see Eqn. 17

x = system state

ξ_F = flexure modal displacements

ξ_c = control surface displacements

Γ = disturbance input vector

Matrices

$[A_m]$ = aerodynamic coefficient matrix
A = dynamics matrix
B = control distribution matrix
$[C_s]$ = structural damping matrix
E = aerodynamic coefficient error matrix
K = control gain matrix
$[K_s]$ = structural stiffness matrix
$[M_s]$ = structural mass matrix
P_i = eigenvector weighting matrix
Q = state weighting matrix
$[Q_c]$ = calculated unsteady aerodynamic influence coefficient matrix
$[Q_A]$ = s-plane representation of unsteady aerodynamic influence coefficient matrix
R = control weighting matrix
V = matrix whose columns are v_i
W = matrix whose columns are w_i
Λ = diagonal matrix composed of λ_i

Scalars

c = reference chord, 0.75m
H(s)G(s) = loop transfer function
j = $\sqrt{-1}$
k = reduced frequency
L = reference length in gust model, 762m
M = Mach number
q = dynamic pressure
s = Laplace operator
V = forward velocity
β_m = aerodynamic lag frequencies
η = zero mean white noise input to gust model with intensity, $\frac{L}{V}\overline{\xi_G^2}$
λ_i = ith eigenvector
$\overline{\sigma}(E)$ = maximum singular value of E
$\underline{\sigma}(E)$ = minimum singular value of E
ω = circular frequency
ξ_G = normal wind gust velocity
$\overline{\xi}_G$ = rms normal wind gust velocity

Subscripts

i = inboard aileron
o = outboard aileron
ES = eigenspace
LQ = linear quadratic
s = structural

Superscripts

* = complex transpose
T = transpose
-1 = inverse

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Eigenspace Techniques for Active Flutter Suppression

I. Introduction

The objective of the research described in this report is the application of Eigenspace (ES) design techniques to the synthesis of active flutter suppression systems. ES techniques allow the designer to use feedback control to place closed loop eigenvalues and shape closed loop eigenvectors to satisfy performance specifications. The basic theory behind ES design has been given by Moore [1] and others. Moore has shown that it is possible not only to place controllable eigenvalues and shape controllable eigenvectors but also to shape uncontrollable eigenvectors. Since the dynamic response characteristics of a system are determined by its eigenvectors as well as its eigenvalues, the ability to shape eigenvectors provides the designer with an important tool.

If performance specifications are given or can be interpreted in terms of desired closed loop eigenvalues and eigenvectors, ES techniques provide a natural design precedure where the desired eigenstructure (if obtainable) can be calculated directly without iteration. Cunningham [2] has shown how ES techniques can be used to improve aircraft flying qualities by placing rigid body poles in desired locations and decoupling rigid body modes. If performance specifications cannot be stated directly in terms of closed loop eigenvalues and eigenvectors, for

example specifications on rms responses or stability margins, the application of ES techniques is not so straightforward. This is the case in aeroelastic control problems such as flutter suppression. One of the principal contributions of this report is the description of a methodology for application of ES techniques to active flutter suppression and other aeroelastic control problems.

To the authors' knowledge, the full power of ES techniques to shape eigenvectors as well as place eigenvalues has not been applied to active flutter suppression. Ostroff and Pines [3] used eigenvalue placement to design a flutter controller, but did not attempt to shape closed loop eigenvectors. In this report ES design techniques are applied to the design of a flutter suppression system for the DAST ARW-2 flight test vehicle. Only full-state feedback is considered. Use of ES techniques in the design of robust state observers will be the subject of future investigations.

It is shown that ES techniques can easily be used to design a flutter controller which satisfies performance specifications on rms control surface activity at the design condition. This controller also provides some gust load alleviation capability at off-nominal conditions. The ES controller requires no feedback on control surface actuator states and thus the open loop frequency response characteristics of the actuators are retained in the closed loop system.

The remainder of the report is divided into four major sections. First mathematical models of the aircraft, actuators, and normal wind gust are presented and performance requirements are discussed. Next the theory of ES design is described. Then the results obtained by applying this theory to the design of a flutter suppression system for the DAST ARW-2 aircraft are given and compared with results obtained from a controller designed using Linear Quadratic (LQ) optimal control theory. Finally conclusions and suggestions for future research are presented. A brief description and listing of a computer program used to perform ES design is given in Appendix B.

II. Mathematical Models and Performance Requirements

The DAST ARW-2 flight test vehicle is a Firebee II Drone which has been modified by replacing the conventional wing with a high aspect ratio, supercritical wing designed to flutter within the flight envelope. Two control surfaces, an inboard and an outboard aileron, are available on the wing and a stabilizer is available on the horizontal tail. The outboard aileron is to be used for flutter suppression, gust load alleviation, and maneuver load alleviation. The inboard aileron is to be used for maneuver load alleviation, and the stabilizer is to be used to compensate for reduced static stability, for automatic flight control, gust load alleviation, and maneuver load alleviation. In practice only the outboard aileron

is to be used for flutter control. However, both the inboard and outboard surfaces were utilized in the controller designs since ES design methodologies are most useful in the design of controllers for systems with multiple control inputs.

The design flight condition for the flutter control system was a Mach Number of 0.86 and an altitude of 4572 m (15000 ft). At this flight condition the flutter control system was required to stabilize the wing. A normal gust with rms velocity of 3.66 m/s (12 ft/s) was assumed at this condition. The rms deflection of the inboard aileron was limited to 10° and the deflection rate to 130°/s. The rms deflection of the outboard aileron was limited to 15° and the deflection rate to 740°/sec. The control system was also to be evaluated on its ability to reduce bending and torsional stresses and shear forces in the wing at a gust test condition of Mach 0.7 and 4572 m. The vertical gust velocity at this condition was 18 m/s (59 ft/s). In the actual DAST vehicle, the flutter suppression system would not be used at this condition because a separate gust load alleviation system is available. However, the gust test condition did provide a convenient condition for evaluating the performance of the flutter controllers at an off-nominal condition; therefore, in this study it was assumed that the flutter controller was also to be used for gust load alleviation.

Actuator/Control Surfaces

Stabilizer. The stabilizer transfer function was a simple first-order lag

$$\xi_{c_e}/u_e = \frac{20}{s+20} \qquad (1)$$

The allowable rms control surface activity levels for the stabilizer were 7 degrees and 80 degrees/s.

Inboard Aileron. An eleventh-order transfer function with third order numerator dynamics was given for the inboard aileron. Up to about 300 rad/s, a fourth order model gave an acceptable approximation of the eleventh order inboard aileron transfer function. This fourth order approximation was

$$\xi_{c_i}/u_i = 1.614\text{x}10^{11}/(s^2+671s+(477)^2)(s^2+322s+(878)^2) \qquad (2)$$

Outboard Aileron. A seventh-order transfer function with second-order numerator dynamics was given for the inboard aileron. Up to about 300 rad/s, a third order transfer function gave an acceptable approximation of the seventh order outboard aileron transfer function. This third order approximation was

$$\xi_{c_o}/u_o = 1.774\text{x}10^{7}/(s+180)(s^2+251s+(314)^2) \qquad (3)$$

Wind Gust. The wind gust was modeled by the second-order model given below

$$\xi_G/\eta(s) = (1+(\sqrt{3}L/V)s)/(1+(L/V)s)^2 \qquad (4)$$

Aircraft. The modes of the aircraft considered were rigid body plunge and pitch modes and seven symmetric aeroelastic modes of the wing. The flexible aircraft model was

$$([M_s]s^2 + [C_s]s + [K_s]) \quad [\xi_F] + q[Q_c(s)] \quad \begin{bmatrix} \xi_F \\ \xi_c \\ \xi_G \\ \hline V \end{bmatrix} = 0 \quad (5)$$

The aerodynamic influence matrix $[Q_c(s)]$ was calculated over a range of reduced frequencies ranging from 0.0 to 1.2 by a doublet lattice procedure. A rational s-plane approximation of $Q_c(s)$ was given by the approximation

$$[Q_A(s)] = [A_o] + [A_1]\frac{cs}{2V} + [A_2]\left[\frac{cs}{2V}\right]^2 + \sum_{m=1}^{L} \frac{[A_{m+2}]s}{s+\frac{2V}{c}\beta_m}$$

This approximation has been widely used in design of active flutter suppression systems [4-10].

The first column of the matrix $[A_o]$ was set equal to the first column of $[Q_c(0)]$, which is zero since the aerodynamic forces due to plunge displacement are zero. The second column of $[A_o]$ gives the force due to a change in angle of attack resulting from a change in pitch angle and must be set equal to the second column of $[Q_c(0)]$. The first column of $[A_1]$ multiplied by c/2 gives the force due to a change in plunge velocity. Since the force due to a change in angle of attack must be the same regardless of whether this change results from a change in plunge velocity or a change in pitch angle, the first column of

$[A_1]$ must equal the second column of $[Q_c(0)]$ divided by c/2 (there is also a scaling factor of 0.0062486 which must be divided into $[Q_c(0)]$).

Usual procedure is to select the β_m's so as to bracket the reduced flutter frequencies [6]. Once the β_m's are specified, the remaining elements of the $[A_m]$ matrices are determined to give the best least squares fit to Q_c over the range of reduced frequencies for which this matrix has been calculated.

In this study a new method was used to select the β_m's. An error matrix was defined as

$$E(j\omega) = [Q_c(j\omega)] - [Q_A(j\omega)]$$

The norm of this matrix can be bounded above and below by its maximum and minimum singular values [11]

$$\underline{\sigma}(E) \leq \frac{||Ex||}{||x||} \leq \bar{\sigma}(E)$$

The singular values of E are defined as the positive square roots of the eigenvalues of E*E. If the β_m's are chose to minimize $\bar{\sigma}(E)$, the norm of the error matrix will be small. In this study a single β was used in $[Q_A]$. Since the reduced flutter frequency was known to be small (0.15), β was selected to minimize $\bar{\sigma}(E)$ at zero frequency. The procedure for determining the $[A_m]$'s and β was as follows (1) a small initial value for β was arbitrarily selected, (2) the first column of $[A_o]$ was set equal to zero, the second column of $[A_o]$ was set equal to the second column of $[Q_c(0)]$ and the first column of $[A_1]$ was set equal to the second column of $[Q_c(0)]$ divided by c/2 times a scale factor, (3) the remaining values of $[A_o]$, $[A_1]$, $[A_2]$ and $[A_3]$ were determined to give the

best least squares fit to $[Q_c(j2kV/c)]$ over the range of reduced frequencies 0.0, 0.05, 0.2, 0.3, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0 and 1.2 (4) the maximum singular value of the error matrix $E(j\omega=0)$ was calculated (5) β was increased by a small amount and the process was repeated until $\bar{\sigma}(E)$ reached a minimum. The results are summarized below

β	$\bar{\sigma}(E(0))$
0.100	214.4
0.125	138.2
0.130	137.0
0.135	147.0
0.150	162.8

Table 1 Singular Value of Error Matrix versus β

The value of β equal to 0.13 which minimizes $\bar{\sigma}(E(0))$ is very close to the reduced flutter frequency of 0.15. The eigenvalues resulting from this model for a Mach number of 0.86 and an altitude of 4572 m are given below

Mode	Eigenvalues
plunge	0, -1.17
pitch	-1.74±j6.29
1st flexure	41.6±j118.1
3rd flexure	-0.7±j136.6
4th flexure	-97.1±j108.4
6th flexure	-16.9±j218.3
8th flexure	-4.1±j397.4
9th flexure	-11.0±j425.0
10th flexure	-24.8±j452.5

Table 2 Eigenvalues of DAST ARW-2 Flight Test Vehicle at M=0.86, h=4572m based on 2-Rigid Body and 7 Flexure Modes

The values of the flexure mode eigenvalues are close to those calculated for the ARW-2 using a model containing four lag states [12]. The pitch eigenvalues were not too different from the actual values of $-1.5\pm j5.6$; however, one of the plunge eigenvalues is considerably different from the actual values of $0.0\pm j1.1\times10^{-3}$. The flutter characteristics of the aircraft are not affected by the rigid body modes and these modes are not included in the model used in the control system design. The poorest correlation between elements of the $[Q_C]$ and $[Q_A]$ matrices occurred for the coefficients associated with the gust velocity, even then rms structural and control surface gust responses calculated using the model with a single β were close to those calculated from higher-order models using several values of β_m.

It is felt that selection of the numerical values of the β_m's based on the maximum and minimum singular values of an error matrix such as given by Eq. 7 has considerable potential in generating low-order approximate models of unsteady aerodynamics. Even the simple approach of minimizing the maximum singular value of the error matrix at a fixed frequency yielded acceptable results. Other approaches such as choosing the β_m's to minimize functions of both the maximum and minimum singular values over a range of frequencies should be investigated.

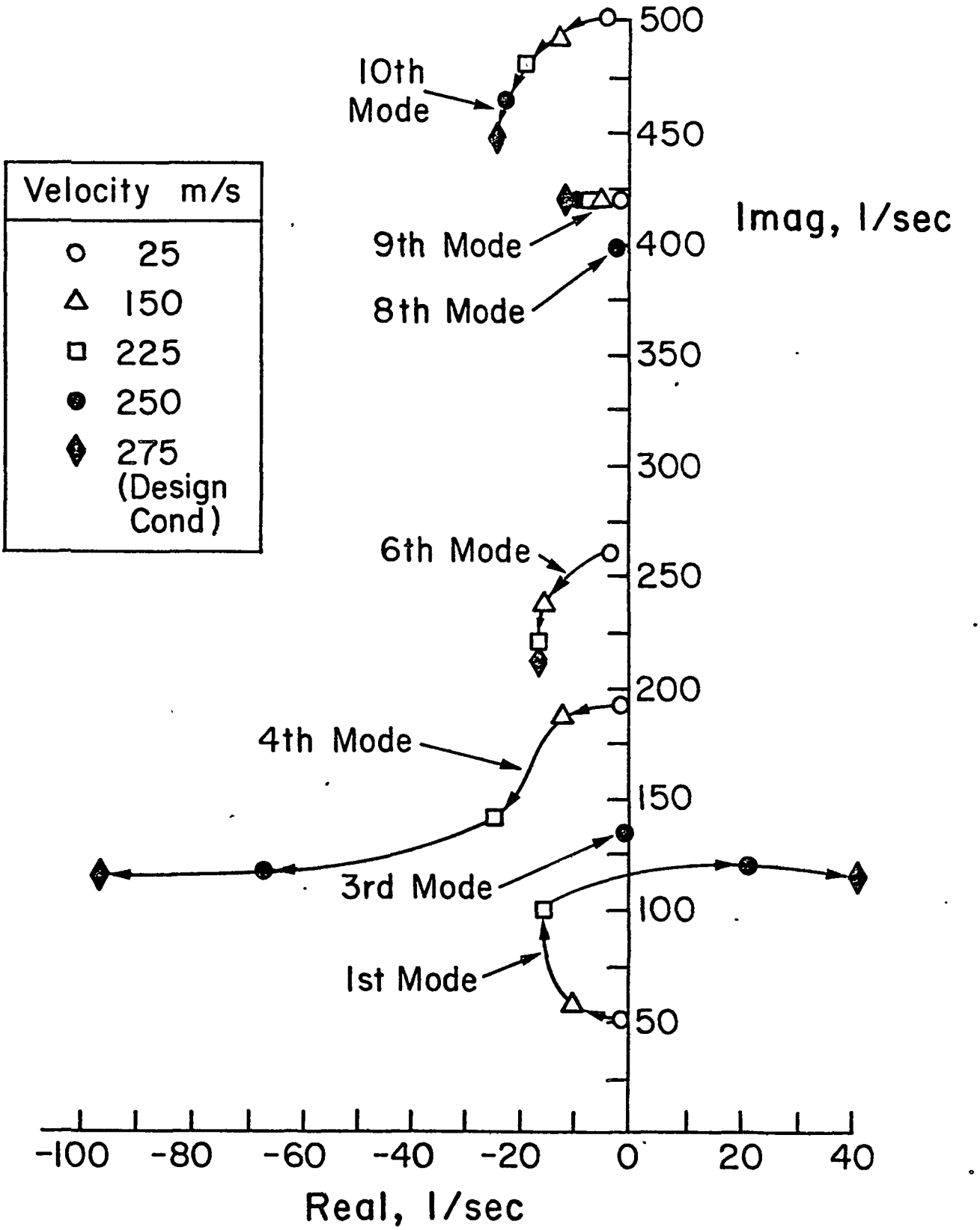


Fig. 1 Root Locus of Seven Mode Model of Uncontrolled Wing as Velocity is Varied

Some possible indices are as follows:

1. Minimize average $\bar{\sigma}$ and $\underline{\sigma}$ over a range of frequencies, i.e.

$$\min\{\frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2}\sigma(E)(j\omega))d\omega\}$$

2. Minimize maximum value of $\bar{\sigma}$ and $\underline{\sigma}$ over a range of frequencies, i.e.

$$\min\{\max(\sigma(E(j\omega))\} \quad \omega_1 \leq \omega \leq \omega_2$$

3. Minimize weighted sum of $\bar{\sigma}$ and $\underline{\sigma}$ over a range of frequencies, i.e.

$$\min\{\sum_{i=1}^{N} a_i\sigma(E(j\omega_i))$$

An examination of the 10 flexure modes (see Appendix A) indicated that modes 2 and 5 were primarily fuselage bending modes and mode 7 was exculsively a tail mode. Therefore these three modes were not considered further in the analysis. Mode 1 was primarily the first wing bending mode, mode 2 was the second wing bending mode, and mode 6 was the first wing torsion mode. These modes were obviously important in modeling flutter and were retained. Modes 3 and 8 included wing tip bending and it was felt that these modes should also be analyzed further. Mode 9 was primarily wing bending and mode 10 was primarily wing torsion and these modes were also retained. Thus seven flexure modes, the 1st, 3rd, 4th, 6th, 8th, 9th and 10th were included in the initial flutter analysis. The loci of the eigenvalues of these modes as velocity was varied are shown in Fig. 1. The first mode flutters at a velocity of about

241 m/s (M=0.75) at a frequency of 120 rad/s. Modes 3 and 8 are insensitive to changes in velocity indicating that they are primarily vibrational modes; however, modes 9 and 10 do vary somewhat with velocity. Eigenvalues were calculated using models in which various modes were deleted. Deletion of the 3rd, 8th, 9th and 10th modes has very little affect on the lower modes.

3 Mode Model	
Mode No	Eigenvalue
6	-15.7±j216.1
4	-94.3±j106.4
1	+39.9±j118.0

Table 3 Eigenvalues of DAST ARW-2 Flight Test Vehicle at M=0.86 h=4572m Based on 3 Flexure Modes.

Basic flutter characteristics could be accurately modeled with only three modes corresponding to the first and second bending (modes 1 and 4) and first torsion (mode 6), and computational expense could be reduced significantly by reducing the order of the system model; therefore, the design studies were based on a structural model containing only these three flexure modes. The loci of the eigenvalues of the three mode model as velocity is varied are shown in Fig. 2.

Equations 2-6 can be combined to give the equations which describe the wing, control surfaces and actuators, and wind gust in vector-matrix form as

$$\dot{x} = Ax + Bu + \Gamma\eta \tag{8}$$

The 18th order state vector, x, consists of (1) the dis-

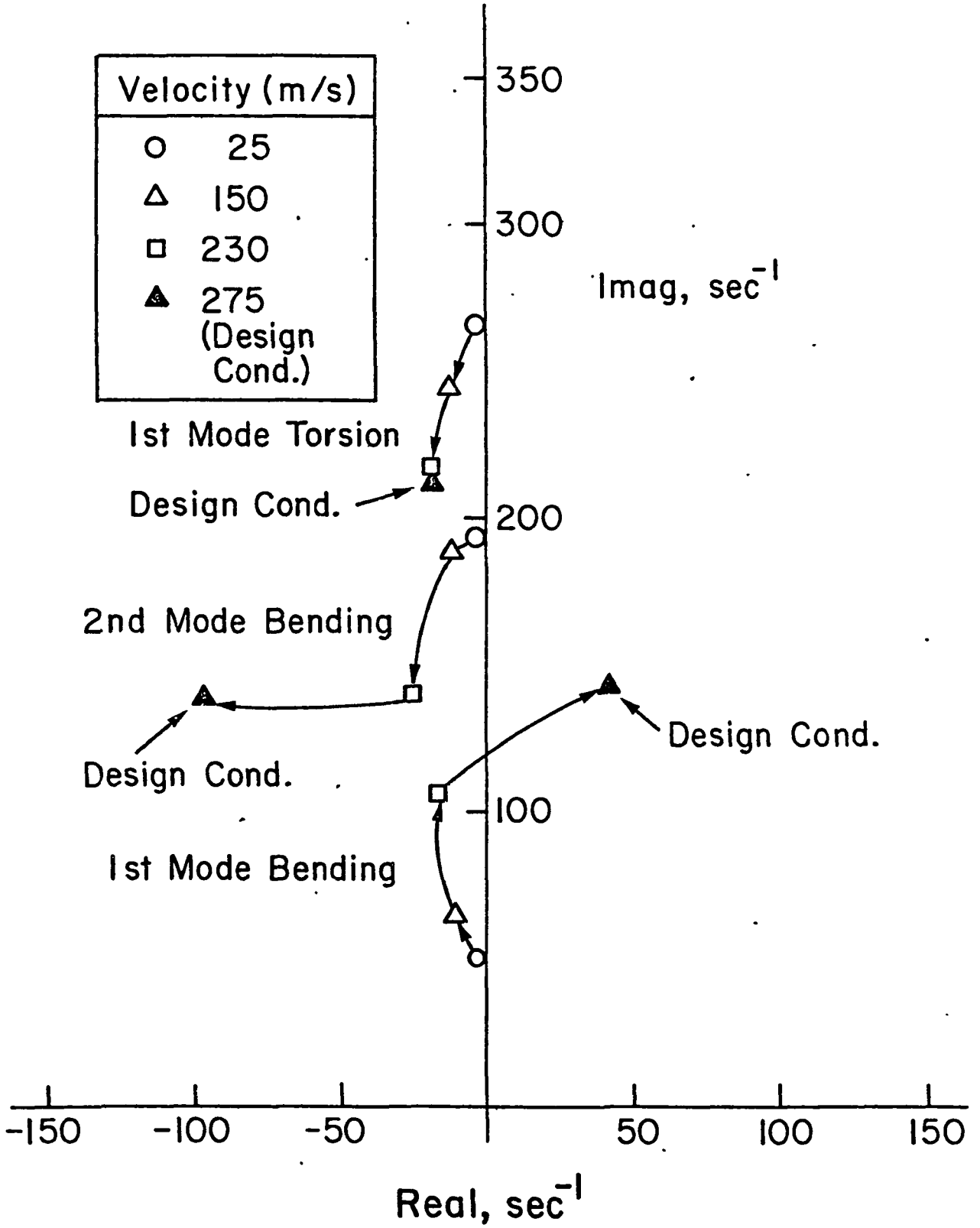


Fig. 2 Root Locus of Three Mode Model of Uncontrolled Wing as Velocity is Varied

placements and velocities associated with the three flexure modes, (2) the unsteady aerodynamic lag states, (3) the states associated with the inboard and outboard control surfaces, and (4) the states associated with the wind gust model. The 2nd order control vector, u, consists of the inputs to the inboard and outboard aileron/actuators and the zero mean white noise input, η, drives the wind gust model. (See Appendix B for a more detailed discussion of Eq. 8)

III. Eigenspace Design

Theory

Moore [1] and others have shown how feedback can be used to directly place eigenvalues and also shape eigenvectors. If performance specifications are given or can be interpreted in terms of desired closed loop eigenvalues and eigenvectors, then ES techniques can provide a natural design procedure. If performance specifications cannot be clearly stated in terms of closed loop eigenvalues and eigenvectors, for example specifications on rms responses, it may be necessary to iterate eigenvalues and eigenvectors until performance specifications are met.

Before discussing the details of the ES flutter controller, the theoretical basis of ES design methodology will be discussed. Consider a system modeled in state variable form as

$$\dot{x} = Ax + Bu + \Gamma\eta$$

with measurements

$$y = Cx$$

where the state $x \epsilon R^n$, the feedback control $u \epsilon R^m$, and the output $y \epsilon R^p$. If we express u in output feedback form

$$u = Ky = KCx$$

the the closed loop system response is

$$\dot{x} = (A + BKC)x + \Gamma\eta \tag{9}$$

The ES design procedure consists, of determining a gain matrix, K, such that for all desired closed loop eigenvalue (λ_i) and eigenvector (v_i) pairs

$$(A + BKC)v_i = \lambda_i v_i \tag{10}$$

Introduce $w \epsilon R^m$ where

$$w_i = KCv_i \tag{11}$$

The problem becomes one of determing w_i such that

$$(\lambda_i I - A)v_i = Bw_i$$

where K is determined from Eq. 11.

Case 1: Full State Feedback

If all the states of the system are available, the feedback control law becomes (assuming p=n and letting C=I without loss in generality)

$$u = Kx$$

The design problem is to find w_i and K such that

$$(\lambda_i I - A)v_i = Bw_i \tag{12}$$

and

$$w_i = Kv_i \tag{13}$$

For the entire collection of closed loop eigenvalues and eigenvectors Eq. (12) becomes

$$V\Lambda - AV = BW \tag{14}$$

Likewise from Eq. (13)

$$W = KV \tag{15}$$

therefore

$$K = WV^{-1} \tag{16}$$

Case 2: Output Feedback (p<n)

$$y = Cx$$

In this case Eq. (15) becomes

$$W = KCV$$

Since C is of rank p<n a unique solution for K is impossible. The alternative is to place only p eigenvalue-eigenvector pairs, i.e. choose p λ_i's and the corresponding p columns of V and solve for K such that

$$K = W(CV)^{-1}$$

The key design issue is to find W which satisfies Eq. (14). In general one cannot completely satisfy both exact eigenvalue and eigenvector placement.

Case 3: Single Input Systems

For single inputs Eq. (12) reduces to

$$(\lambda_i I - A)v_i = bw_i$$

where w_i is a scalar. This single variable cannot be adjusted to place n parameters on the left hand side, therefore, the single input case only involves pole placement with arbitrary eigenvector position. The gain K is unique in this case.

Case 4: Multiple Input Systems

The real benefit of eigenspace techniques is realized when more than one control is available. If the rank of B

is n, i.e., one independent control for each state then from Eq. (12)

$$w_i = B^{-1}(\lambda_i I - A)v_i$$

for each desired λ_i and v_i of the closed loop system.

Practically speaking, however, one has fewer controls than states and exact placement of λ_i and v_i for all i=1,2,...n is impossible. The design procedure then becomes one of choosing portions of v_i to eliminate certain state responses from a mode while emphasizing others and letting other responses react arbitrarily. Assuming the system is controllable and rank (B) = m<n, only m free parameters can be specified. If it is desired to change an eigenvalue associated with a controllable mode, then for the new eigenvalue, λ_i, the matrix $(\lambda_i I - A)$ is nonsingular. Thus

$$v_i = (\lambda_i I - A)^{-1} B w_i \tag{17}$$

or

$$v_i = L_i w_i \tag{18}$$

where

$$L_i = (\lambda_i I - A)^{-1} B$$

Since there are not enough independent controls available to arbitrarily place all λ_i's and v_i's, the w_i's are selected to minimize the following least square performance index

$$J_i = (v_{d_i} - v_i)^* P_i (v_{d_i} - v_i)$$

where P_i is used to emphasize certain components of v_{d_i}. Solving for w_i which minimizes J_i produces an optimal psuedo-

inverse

$$w_i = (L^*_i P_i L_i)^{-1} L^*_i P_i v_{d_i}$$

and v_i is given by Eq. (18).

If certain eigensolutions are not to be altered (e.g. actuator poles) or if λ_i is uncontrollable then

$$w_i = 0$$

and Eq. (12) is satisfied. The general design procedure used is to vary P_i, λ_i, and v_{d_i} until performance specifications on rms values of the state and control are met.

The above analysis is valid for complex or real eigensolutions. If an entirely real K matrix is desired then the the problem must be decoupled into its real and imaginary parts. If

$$\lambda = \lambda_R + j\lambda_I$$

$$v = v_R + jv_I$$

$$w = w_R + jw_I$$

then for real A

$$(\lambda_i I - A) v_i = B w_i$$

becomes in entirely real terms

$$\begin{bmatrix} (\lambda_R I - A) & -\lambda_I I \\ \lambda_I I & (\lambda_R I - A) \end{bmatrix} \begin{bmatrix} v_R \\ v_I \end{bmatrix} = \begin{bmatrix} Bw_R \\ Bw_I \end{bmatrix} \qquad (19)$$

and

$$\begin{bmatrix} v_R \\ v_I \end{bmatrix} = \begin{bmatrix} (\lambda_R I - A) & -\lambda_I I \\ \lambda_I I & (\lambda_R I - A) \end{bmatrix}^{-1} \begin{bmatrix} Bw_R \\ Bw_I \end{bmatrix} \qquad (20)$$

or
$$\begin{bmatrix} v_R \\ v_I \end{bmatrix} = L \begin{bmatrix} w_R \\ w_I \end{bmatrix}$$

where
$$L = \begin{bmatrix} (\lambda_R I - A) & -\lambda_I I \\ \lambda_I I & (\lambda_R I - A) \end{bmatrix}^{-1} \begin{bmatrix} B & O \\ O & B \end{bmatrix} \tag{21}$$

Then the optimal psuedo-inverse solution in entirely real terms is

$$w_i = (L_i^T P_i L_i)^{-1} L_i^T P_i v_{d_i} \tag{22}$$

where
$$w_i = \begin{bmatrix} w_{R_i} \\ w_{I_i} \end{bmatrix}$$
$$v_{d_i} = \begin{bmatrix} v_{d_{R_i}} \\ v_{d_{I_i}} \end{bmatrix}$$
$$P_i = \begin{bmatrix} P_{R_i} & O \\ O & P_{I_i} \end{bmatrix}$$

To determine K with λ_1 and λ_2 complex conjugates, we must solve

$$K[v_{R_1}+jv_{I_1}, v_{R_1}-jv_{I_1}, V] = [w_{R_1}+jw_{I_1}, w_{R_1}-jw_{I_1}, W] \tag{23}$$

where V and W are the remaining v_i's and w_i's. Post multiplying both sides of this equation by

$$R = \begin{bmatrix} \frac{1}{2} & -j\frac{1}{2} & 0 \\ \frac{1}{2} & +j\frac{1}{2} & \\ 0 & & I \end{bmatrix} \tag{24}$$

yields

$$K[v_{R_1}, v_{I_1}, V] = [w_{R_1}, w_{I_1}, W] \quad (25)$$

Now the left hand eigenvector matrix is nonsingular so

$$K = [w_{R_1}, w_{I_1}, W][v_{R_1}, v_{I_1}, V]^{-1} \quad (26)$$

This procedure can be applied to any complex conjugate pair. It should be noted, however, that the transformation matrix R is not unique so neither is K.

Application to Active Flutter Suppression

Using the above procedure a flutter control system was designed. The initial ES controller was designed by rotating the unstable eigenvalues around the imaginary axis and leaving all other eigensolutions in their open loop configuration. This resulted in acceptable rms control surface activity at the flutter condition and a stable response at the gust test condition. Although the ES design stabilized the wing at the gust test condition, the maximum allowable values for the rms inboard deflection rate and outboard deflection and deflection rate were exceeded. It was felt that the performance at the gust test case might be enhanced by redesign of the control system. Since the aircraft exhibits satisfactory response at velocities somewhat less than the flutter speed, it was decided to use ES design techniques to force the closed loop eigenvalues at the design velocity (275 m/s) to be the same as the open loop eigenvalues at the velocity of 200 m/s (this is 20% less than the flutter speed) and to force the closed

loop eigenvectors at 275 m/s to approach the open loop eigenvectors at 200 m/s. The control was designed to retain the eigenvalues and eigenvectors associated with the actuators, unsteady aerodynamics, and gust model at their open loop values so that energy would not be transferred to these modes. (The gust states are uncontrollable and the gust eigenvalues cannot, in any case, be moved.) The ability to keep poles in desired locations is one of the key advantages of the ES design.

In the open loop condition, the actuator dynamics are decoupled from the aeroelastic modes; therefore, the components of the aeroelastic eigenvectors in the direction of the actuator states are zero. Thus driving the closed loop aeroelastic eigenvectors to exactly their open loop values would decouple the actuators from the aeroelastic response of the wing. This is obviously not desirable since the wing would be uncontrolled; however, it is possible to reduce control surface activity and still stabilize the wing by using the weighting matrices P_i to penalize large values of the components of the closed loop aeroelastic eigenvectors in the actuator directions. Initially all closed loop aeroelastic eigenvalues at 275 m/s were placed at the locations of the open loop aeroelastic eigenvalues at 200 m/s and the weighting matrices were chosen to be identity matrices. This resulted in a reduction of control surface activity of less than 7% at the gust test condition. The rms inboard aileron deflection rate was still over three

times its allowable value while the outboard aileron deflection and rates were about twice their allowable values. A gust of 5.7 m/s would saturate the inboard control surface while the outboard surface remained unsaturated since its deflection and rate were about two-thirds of the allowable values.

It was decided to shift some of the control effort from the inboard to the outboard aileron in an attempt to increase the gust velocity at which the system would saturate. This was accomplished by increasing the weights on the components of the aeroelastic eigenvectors in the inboard actuator directions while retaining all other weights at one. Values of these weights were increased to 2.5×10^3 yielding the results in Table 4. The inboard rate was reduced substantially without excessively increasing outboard activity. Specifications on inboard rate and outboard deflection and rate were still not met at the gust test condition but all of the quantities were about twice their maximum values. Thus both control surfaces would saturate at about the same gust velocity (8.3 m/s). Since it proved impossible to further reduce outboard surface activity by adjusting the weighting matrices P_i, the ES controller resulting from eigenvector shaping to reduce inboard rate was chosen for further evaluation.

Table 4

Comparison of RMS Control Surface Activity
for Various Controller Designs and Flight Conditions

Controller Design	*Inbd Defl (Deg)*		*Inbd Rate (Deg/s)*		*Outbd Defl (Deg)*		*Outbd Rate (Deg/s)*	
	Flutter	*Gust Test*	*Flutter*	*Gust Test*	*Flutter*	*Gust Test*	*Flutter*	*Gust Test*
Unstable Roots Rotated About Imag. Axis								
ES	0.5	5.0	108.0	412.0	3.3	31.4	509.0	1464.0
Unstable Roots Rotated About Imag. Axis								
LQ	1.8	*	271.0	*	2.8	*	407.0	*
Eigenvector shaping to Reduce Inbd Rate (Final ES)	0.9	8.1	86.0	279.7	4.7	31.2	612.0	1464.0
Weighting Inbd Rate in Perf Index (Final LQ)	0.7	4.6	85.0	284.1	3.1	30.2	486.0	1335.1
Final ES Inbd Failed	-	-	-	-	4.4	26.5	572.0	1427.0
Final LQ Inbd Failed	-	-	-	-	3.5	30.0	519.0	1410.0
Max Allowable	10.0	10.0	130.0	130.0	15.0	15.0	740.0	740.0

*Unstable

Results

In addition to the ES design, a flutter controller was designed using Linear Quadratic(LQ) optimal control theory. LQ control theory has been discussed extensively in numerous texts, for example Ref. 13. The basic LQ design procedure consists of selecting quadratic weighting matrices Q and R in a scalar performance index

$$J = \tfrac{1}{2} \int_0^\infty [x^T Q x + u^T R u] dt \qquad (27)$$

The control which minimizes this performance index is given as

$$u = Kx$$

where K is the solution of a matrix Ricatti equation.

It is well known that if Q is set equal to zero and R equal to the identity matrix in the performance index given by Eq. (27), then all stable eigenvalues will remain unchanged and all unstable eigenvalues will be rotated about the imaginary axis [13]. Initial LQ design was performed using this approach; however, as shown in Table 4, the rms deflection rate for the inboard actuator was approximately double its allowed maximum at the flutter test condition and the controller resulted in an unstable wing at the gust test condition (note the uncontrolled wing is stable at this condition). Since all rms responses except the inboard actuator deflection rate were acceptable at the flutter test condition, only this state was weighted in the performance index. The results of varying the weight on inboard actuator deflection rate is shown in Table

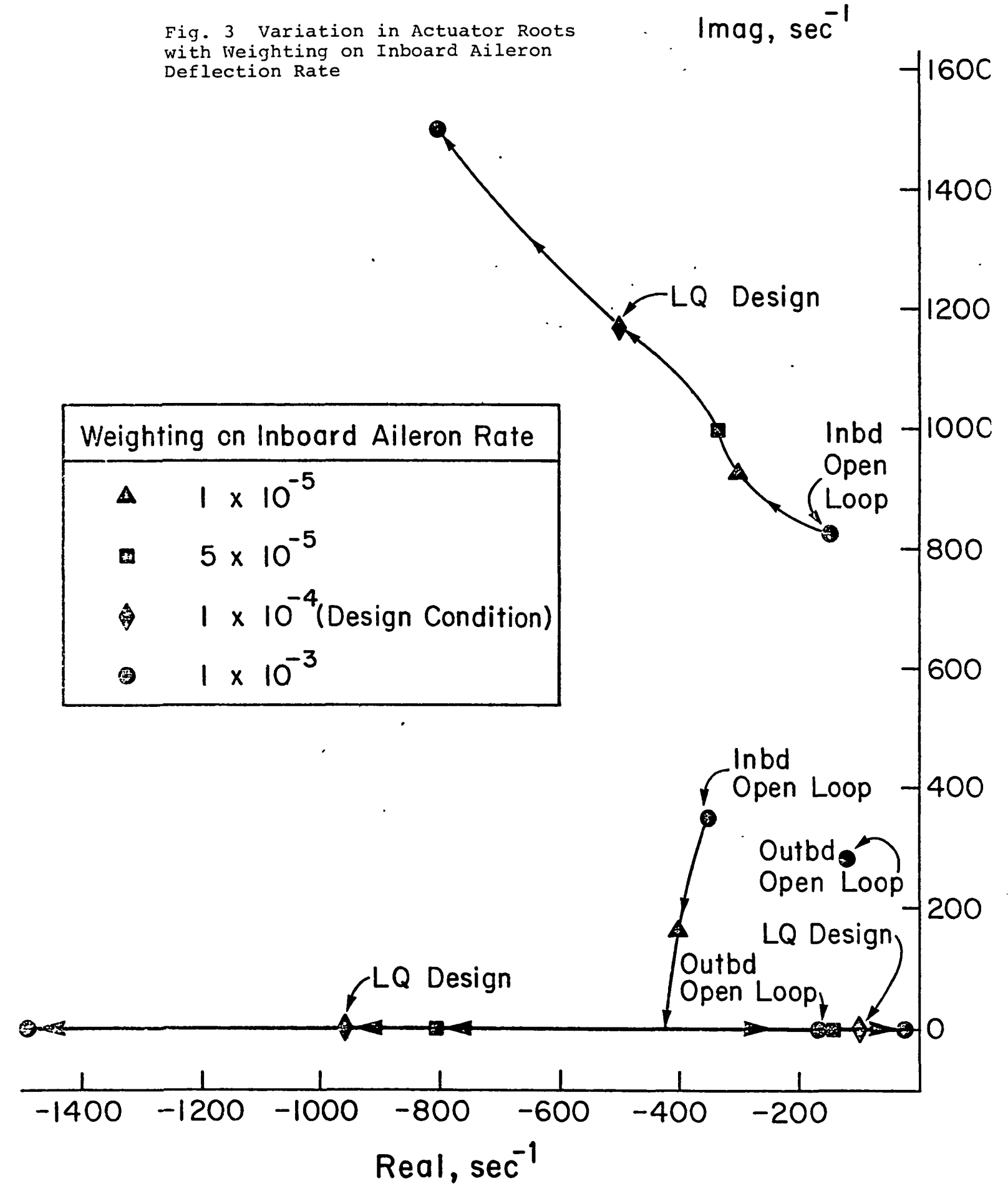
Table 5

RMS Response for Various Weights
on Inboard Actuator Rate

Weight on Inbd Deflection Rate	RMS Response (Degrees and Degrees/s)			
	Inbd Def.	Inbd Rate	Outbd Def.	Outbd Rate
0	1.60	217.6	2.73	427
1×10^{-5}	1.58	256.0	2.74	428
5×10^{-5}	0.92	122.0	3.02	470
1×10^{-4}	0.66	84.8	3.14	486
5×10^{-4}	0.22	26.0	3.32	514
1×10^{-3}	0.12	14.0	3.36	518
Max Allowable	10.0	130.0	15.0	740

5. The rms value of the inboard actuator deflection rate decreases fairly rapidly as its weight is increased and the rms activity level of the outboard control surface does not increase substantially. The eigenvalues associated with the flexure modes and the outboard aileron do not change as the weight on the inboard actuator rate is changed; however, as shown in Fig. 3, the eigenvalues associated with the inboard actuator change radically. The moduli of three of the roots become very large while the fourth root approaches zero. Since any large change in actuator frequency response characteristics would be difficult to obtain without substantially redesigning the actuator, a value for the weight on actuator deflection rate of 1×10^{-4} was selected. This gave acceptable rms responses at the flutter test condition and also stabilized the wing at the gust test condition and did not affect the frequency response of the inboard actuator too much. The open and closed loop inboard actuator frequency response curves for the final design are shown in Fig. 4. It can be seen that the overall gain for the closed loop system is reduced compared with the open loop response. At zero frequency the open loop gain is unity and the closed loop gain is 0.79. Thus the forward loop gain would have to be increased by 21% in order to restore the steady state gain to its open loop value. This might be difficult without modifications to existing actuator hardware.

Fig. 3 Variation in Actuator Roots with Weighting on Inboard Aileron Deflection Rate



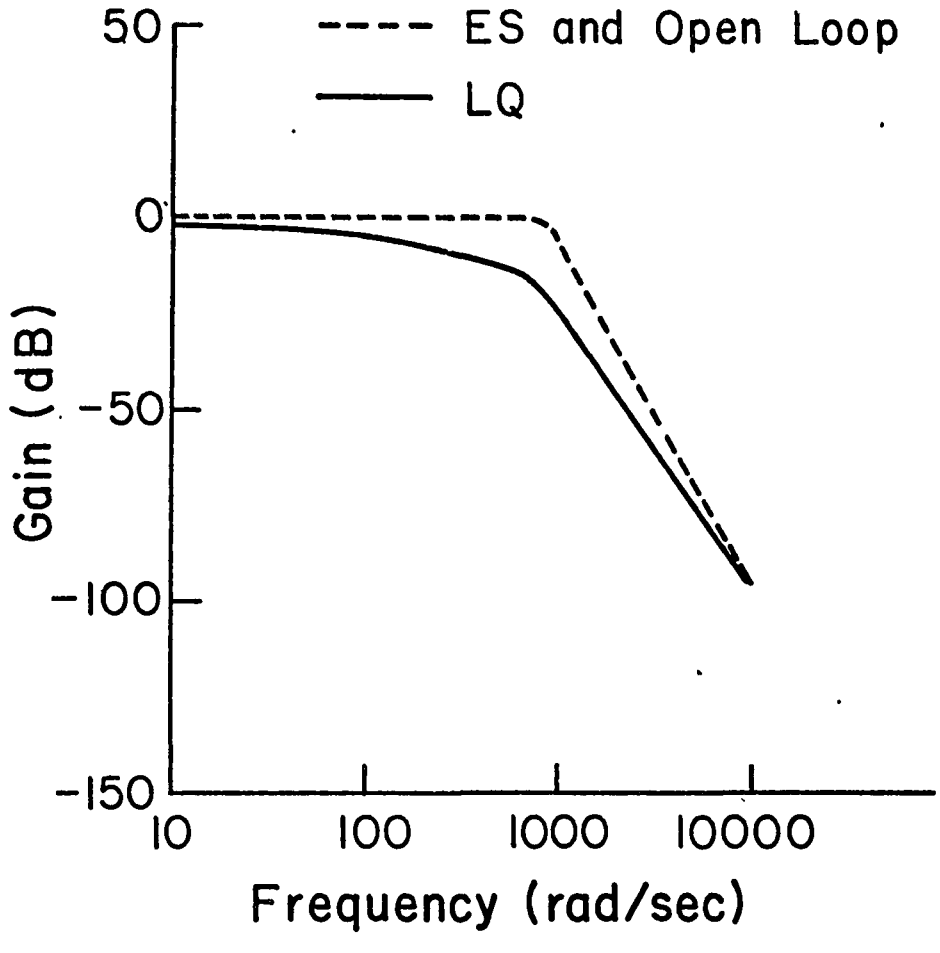


Fig. 4 Inboard Actuator Transfer Function Frequency Response

It appears to be impossible to design either an ES or LQ flutter controller which also meets the specification on rms control surface activity at the gust test condition. In the actual DAST ARW-2 vehicle, the inboard aileron is used for manuever load alleviation but not for flutter suppression or gust load alleviation. Furthermore a different controller design is used for gust load alleviation than for flutter suppression. Thus it is not too surprising that the ES and LQ flutter controllers do not meet specifications on rms surface activity at the gust test condition. However, if the rms gust velocity at the gust condition is reduced to 8.3 m/s (about 50% of its nominal value) both the ES and LQ controllers meet all specifications on rms surface activity. Table 6 shows that both controllers provide some torsional load alleviation at this flight condition. The bending and torsional stresses and shear force were calculated at nine stations on the wing (station 1 is at the root and station 9 is near the tip). The LQ design results in the largest rms bending moments at all stations. The ES design results in bending moments which are lower than those resulting from the LQ design but higher than the uncontrolled values. The differences in bending moment in all three cases are not large. Both the LQ and ES designs result in substantial reductions (over 50%) in torsional stresses at all stations compared with the uncontrolled values. The LQ reduces torsional stresses slightly more than the ES design. The ES design results in lowest values of shear near the wing

Table 6

RMS Loads at Modified Gust Test Condition
(M=.7, h=4572m, V_G=8.3 m/s)

		Root			Station					Tip
		1	2	3	4	5	6	7	8	9
FINAL LQ	*Bending Moment (N-M)*	5740	4530	3426	2444	1371	889	377	83	5
	Shear (N)	92	82	74	63	42	36	24	6	1
	Torque (N-M)	36	54	73	82	73	63	52	6	2
FINAL ES	*Bending Moment (N-M)*	5558	4412	3359	2418	1374	900	389	91	5
	Shear (N)	87	79	71	61	42	36	24	6	1
	Torque (N-M)	46	63	79	85	76	65	53	7	1
OPEN LOOP	*Bending Moment (N-M)*	5545	4262	3116	2125	1115	671	248	47	3
	Shear (N)	98	85	75	61	38	31	18	3	1
	Torque (N-M)	106	130	152	156	139	121	101	24	3

root but the uncontrolled values of shear are less near the tip. The differences in shear between the LQ, ES, and uncontrolled cases are not large.

The root locus of the eigenvalues of the wing with the LQ controller is shown in Fig. 5 as velocity is varied. The wing goes unstable at a velocity of about 295 m/s (the design condition was 275 m/s and the uncontrolled flutter speed was 241 m/s). It is interesting to note that for the LQ controller, one of the roots associated with the unsteady aerodynamic lag states goes unstable resulting in zero frequency flutter. In the uncontrolled case, the first bending model goes unstable in classical coupled mode flutter (Fig. 1). The root locus for the ES design is shown in Fig. 6. For the ES controller the wing goes unstable at a velocity of about 310 m/s. As in the uncontrolled case the first bending mode goes unstable, but in the controlled case the eigenvalues associated with this mode move to the real axis where one real root goes unstable resulting in zero frequency flutter.

The results of varying altitude while maintaining Mach number constant are shown in Fig. 7. At M=0.86, the uncontrolled wing is unstable until an altitude of 6700 m is reached. At the same Mach number, the LQ controller results in a stable wing at all altitudes above 3200 m and the ES controller stabilizes the wing above altitudes of 2900 m. At M=0.7 the uncontrolled wing is stable for altitudes above 1800 m whereas the ES controller stabilizes the

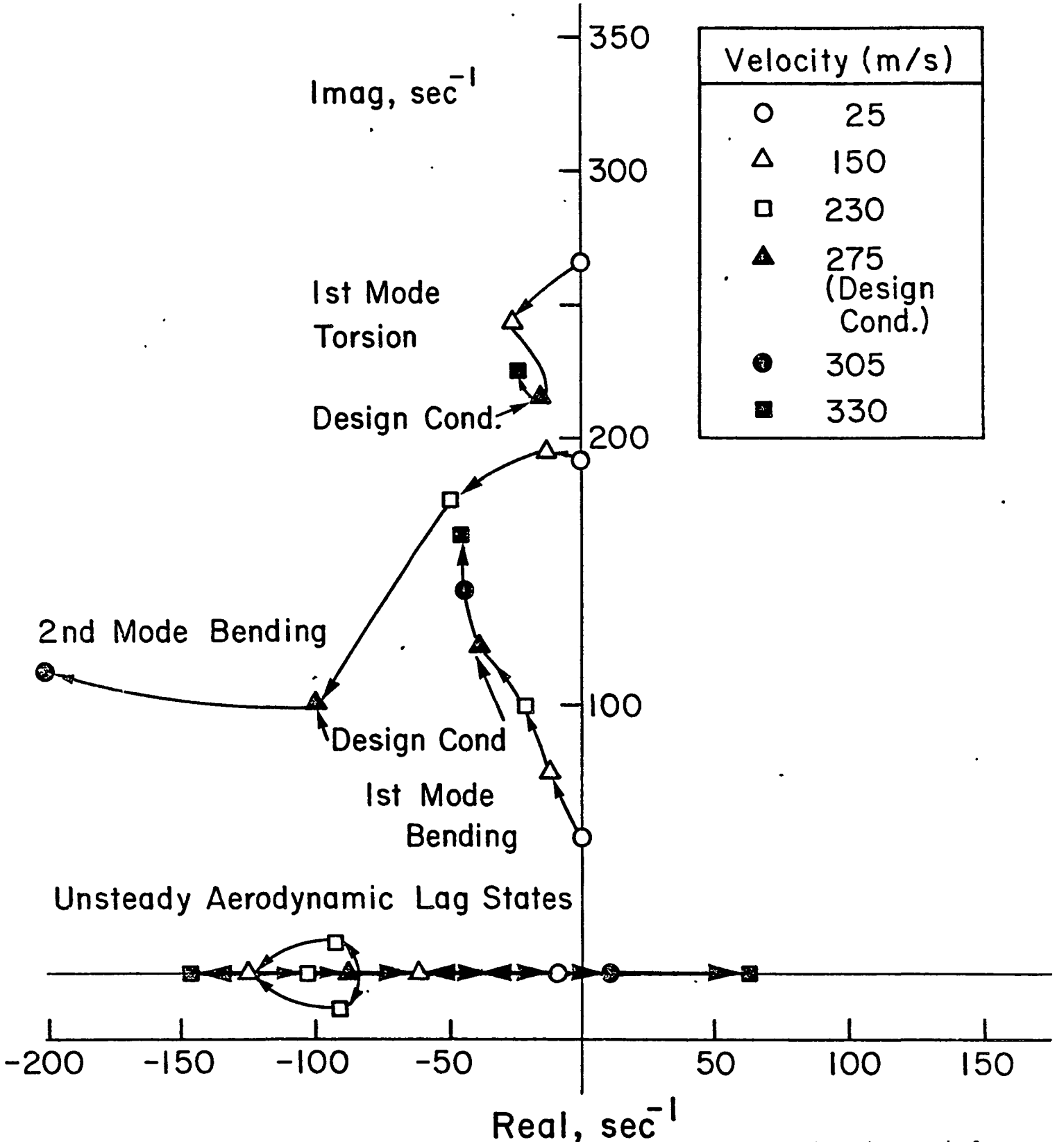


Fig. 5 Root Locus of LQ Controlled Wing as Velocity is Varied

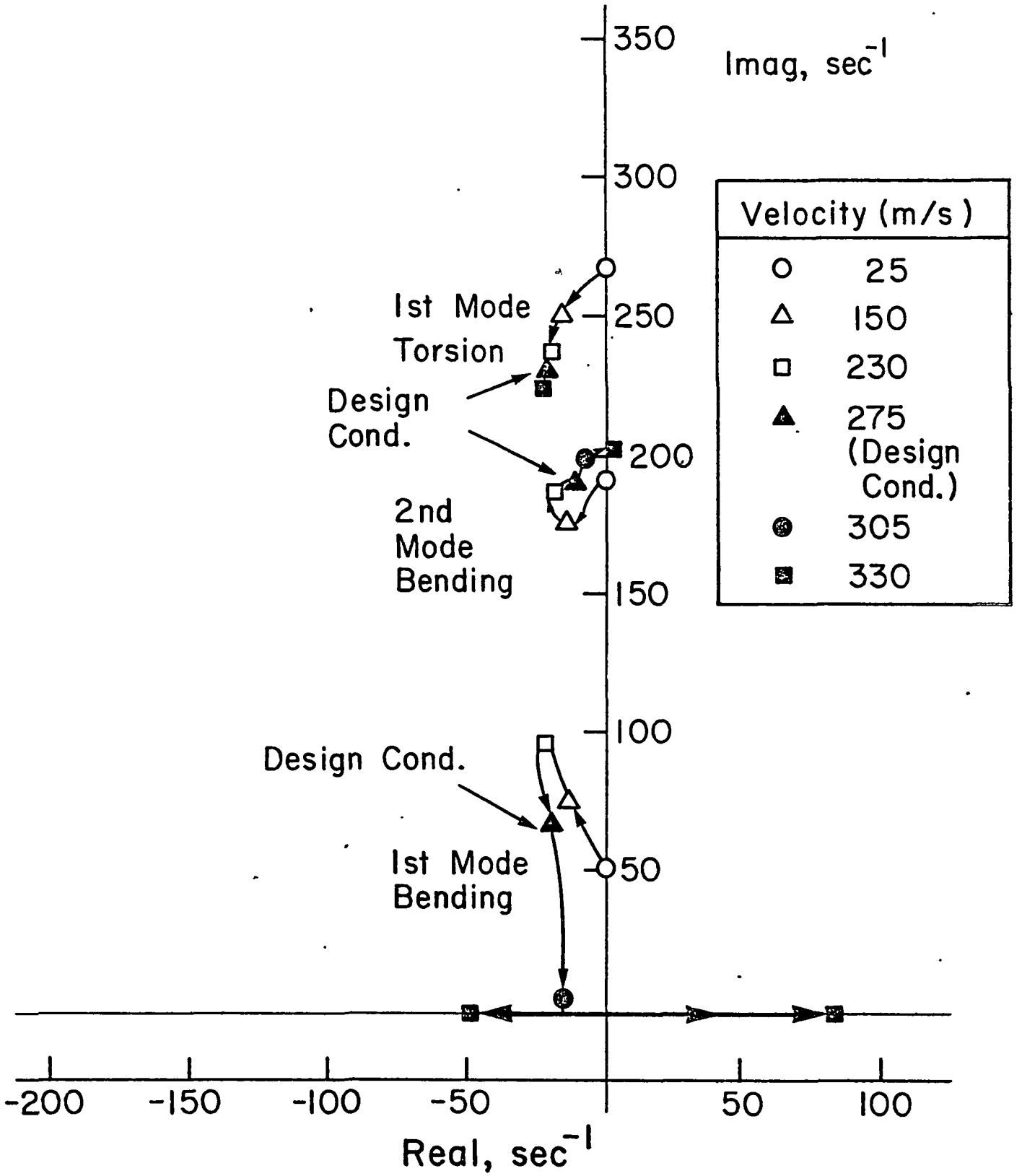


Fig. 6 Root Locus of ES Controlled Wing as Velocity is Varied

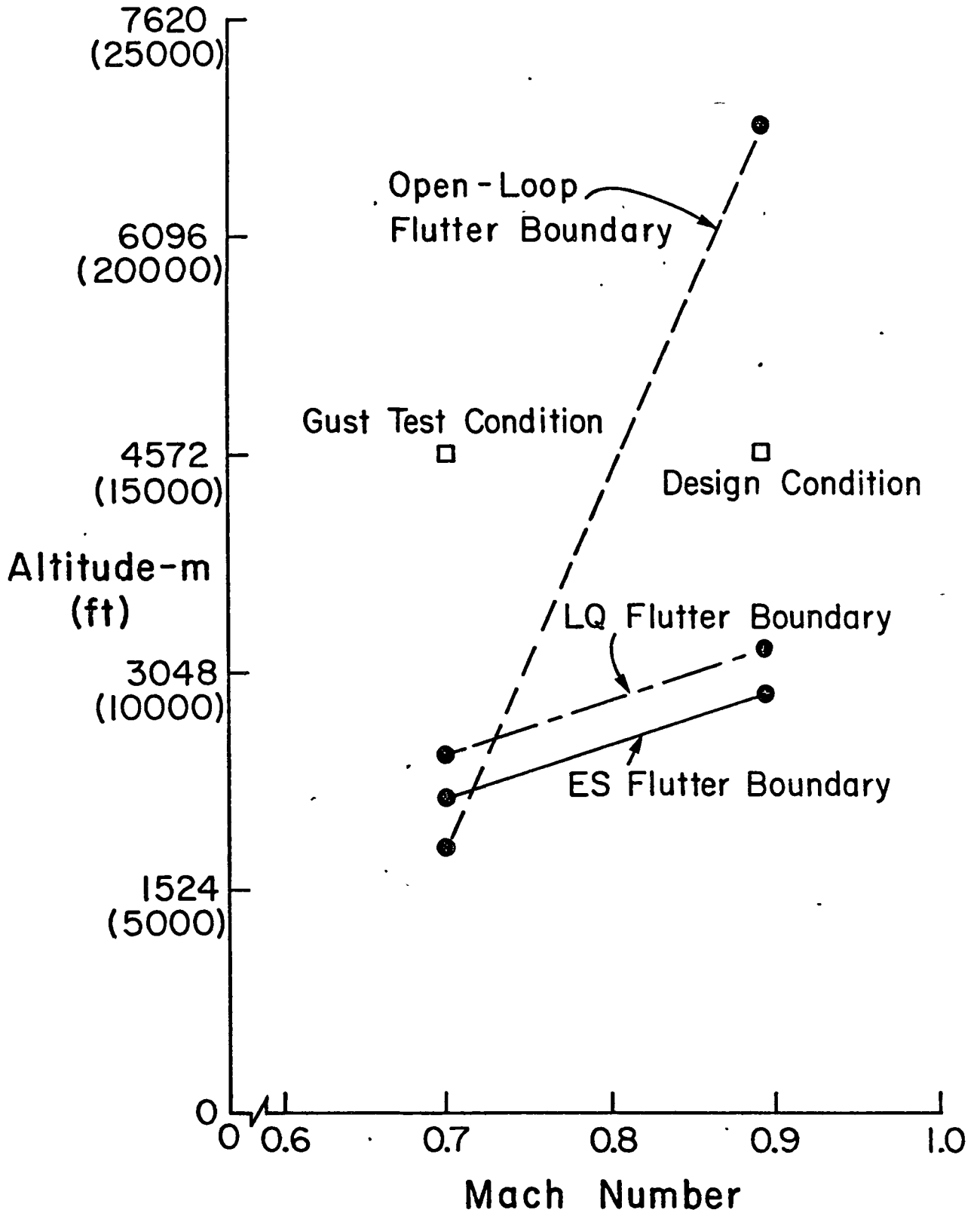


Fig. 7 Flutter Boundaries for Uncontrolled, LQ, and ES Controlled Wing

wing for altitudes above 2100 m and the LQ controller stabilizes the wing for altitudes above 2400 m.

The ability of the ES and LQ systems to stabilize the wing in case of an actuator failure yielded little difference in the performance of the two systems. In the case where an inboard actuator failure was simulated, the outboard aileron was capable of stabilizing the wing with only small increases in rms surface activity (see Table 4). When the outboard actuator failed the inboard aileron was unable to stabilize the wing in both designs.

Since the outboard aileron is critical in stabilizing the wing, it is important to examine the stability margins associated with the outboard control loop. This can be accomplished by examining the loop transfer function

$$H(s)G(s) = K^T[Is-A]^{-1}B$$

Note since $u = +Kx$, the characteristic equation is given by

$$1 - G(s)H(s) = 0$$

and the critical condition occurs when the phase angle of $H(j\omega)G(j\omega)$ is zero. For the LQ design

$$H(s)G(s)_{LQ} = \frac{-148.8(s-2.6)}{(s^2-79.8s+(124.5)^2)}$$

It is interesting to note that the actuator poles, the poles associated with the unsteady aerodynamic lag states and all of the poles associated with the stable flexure modes have been cancelled by zeros. The only remaining poles are those associated with the unstable first bending mode. Bode diagrams for the LQ loop transfer function are

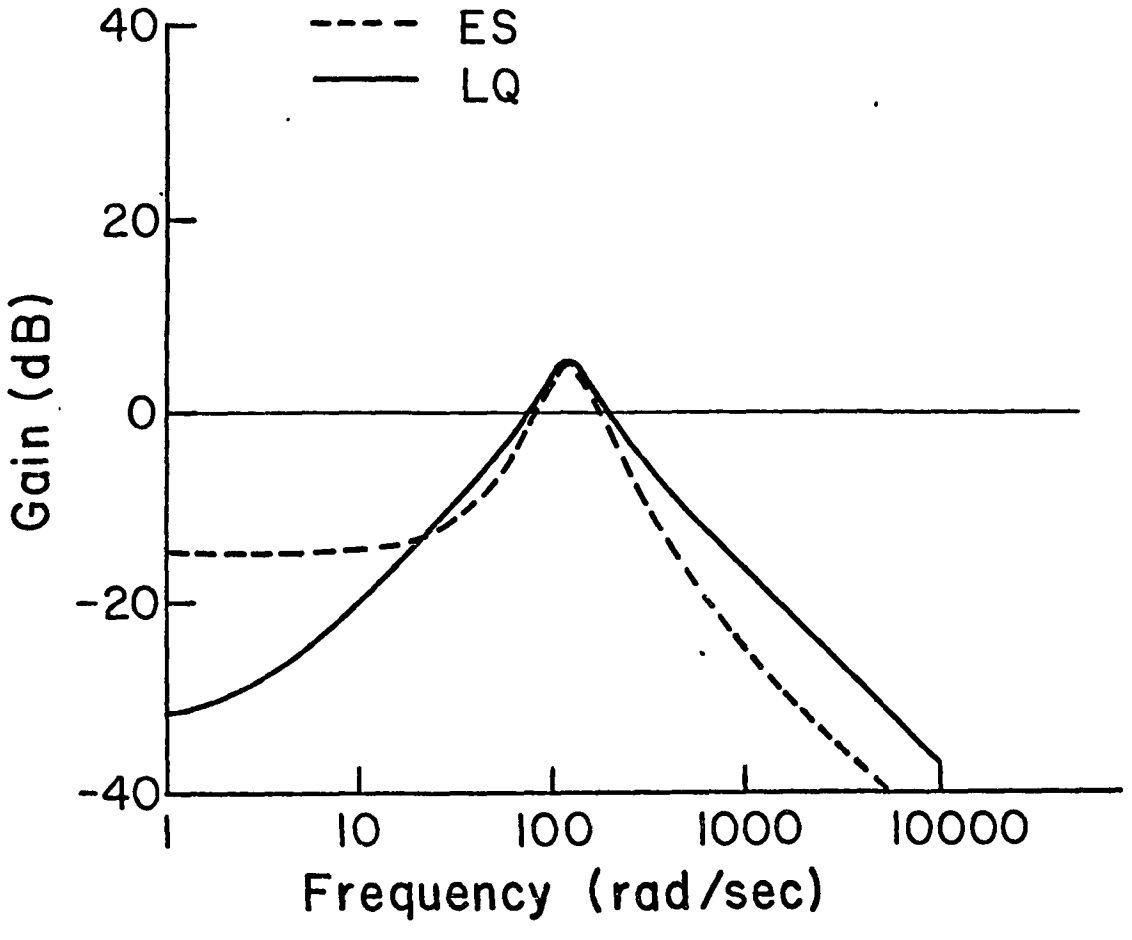


Fig. 8 Frequency Response of Magnitude of H(s)G(s) for LQ and ES Designs

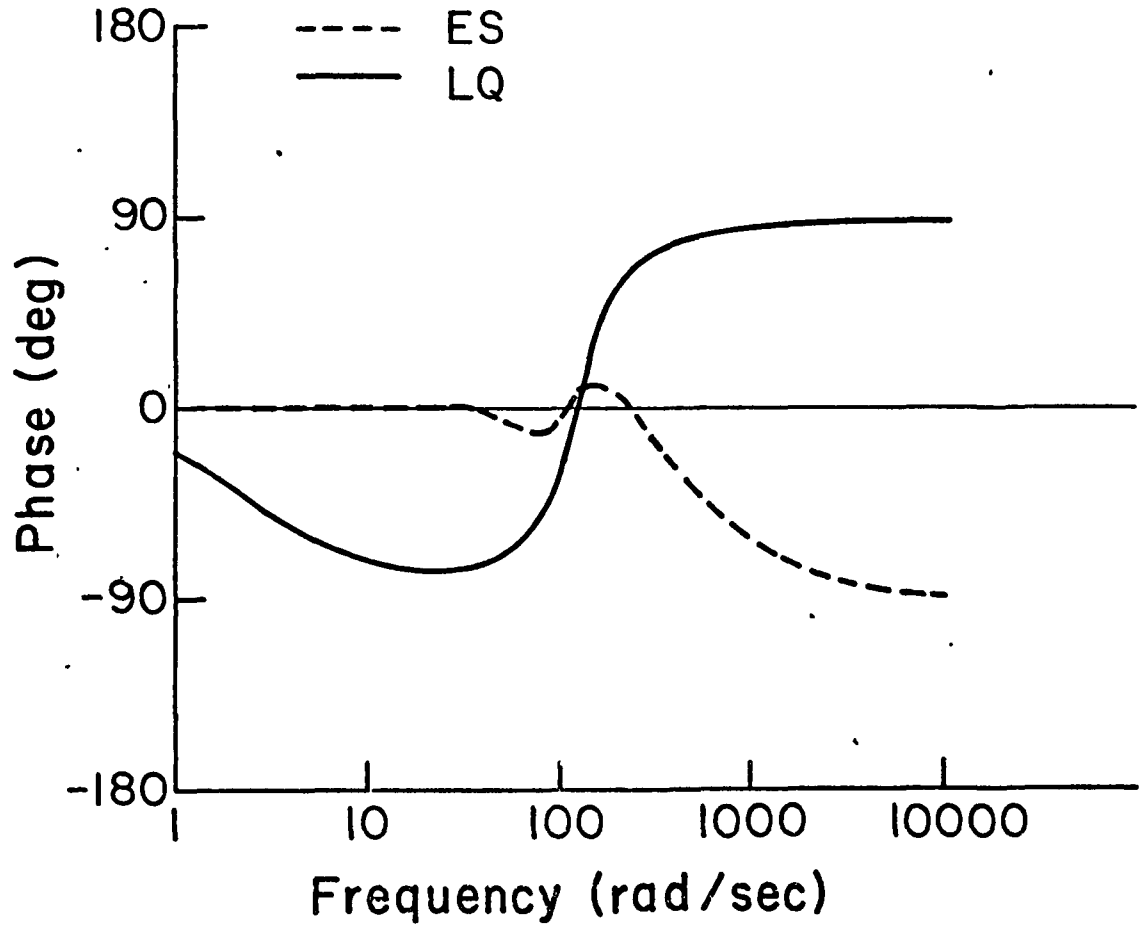


Fig. 9 Frequency Response of Phase of H(s)G(s) for LQ and ES Designs

given in Figs. 8 and 9. The gain margin is 6Db and the phase margins are 60°. This is not surprising, however, since Safanov and Athans [14] show that LQ controllers yield excellent gain and phase margins.

The loop transfer function for the ES controller is

$$H(s)G(s)_{ES} = \frac{51.45(s-427.4)(s-66.5)(s+42.4)}{(s^2-79.8s+(124.5)^2)(s^2+188.6s+(142.2)^2)}$$

Again all actuator poles and poles associated with unsteady aerodynamic lag states have been cancelled by zeros. But only the poles associated with the first torsion mode have been cancelled and the unstable first bending mode and stable second bending mode remain. Bode diagrams for this transfer function are also shown in Figs. 8 and 9. Gain margins are 6 Db or better but phase margins are less than 20°. Gain versus frequency plots for the LQ and ES transfer functions are very similar for frequencies above 10 rad/s. Both curves peak at the flutter frequency with a gain of 6 Db and then roll off with increasing frequency.

Conclusions

The ES and LQ controllers give very similar results in terms of required control surface activity. At the gust test condition the ES design exhibits lower wing root bending moment and shear than the LQ design but the LQ controller provides slightly lower torsional stress. Both the ES and LQ designs provide significantly reduced torsional stress at the wing root, slightly reduced shear, and

slightly increased bending moment compared with the open loop response. Both the LQ and ES controllers significantly increased flutter speed compared with the uncontrolled wing. The ES controller results in a slightly greater flutter speed than the LQ controller. For a fixed Mach number the ES design is stable at lower altitudes than the LQ design. The LQ design exhibits significantly better phase margins than the ES design at the flutter test condition. The gain margins are the same. Since the phase margins are determined near the flutter frequency, it is very important that the ES design be based on a model which is accurate in this frequency range. Both the LQ and ES designs exhibit excellent roll off at higher frequencies where modeling uncertainties are large.

The LQ design requires large feedback gains on the inboard actuator states. This reduces the overall inboard actuator gain. Increased forward loop gain would be required to restore open loop characteristics. This might necessitate redesign of existing actuator hardware. The ES controller does not require actuator feedback and the closed loop frequency response characteristics of the actuators is the same as the open loop case. Since the inboard actuator is not very effective for flutter control this is not an important consideration for the ARW-2, but could be critical in other applications. The ES controller requires only the structural and aerodynamic states, thus a lower-order observer could be used to realize this controller than would be required by the LQ controller. Finally the computational

algorithms required to calculate the ES controller gains are simpler and less expensive to use than those required to calculate the LQ controller gains.

The authors are currently working on ES design techniques that shape eigenvectors associated with uncontrollable states. It appears to be necessary to shape the eigenvectors associated with the gust states in order to further reduce rms control activity. Work is also in progress on improving ES stability margins and realization of ES controllers by means of observers and direct output feedback.

Doyle and Stein [15] have shown that if a Kalman Filter is used for state estimation, the robustness properties of full-state feedback can be recovered by introducing fictitious plant noise in the Ricatti equation used to obtain the filter gain matrix. As the magnitude of this plant noise approaches infinity, the filter poles asymptotically approach the transfer zeros (if the transfer zeros are in the right half of the complex plane the filter poles approach the mirror image of the zeros in the left half plane) or approach infinity in a Butterworth pattern. Since the Doyle-Stein procedure gives the location of the filter poles, it is possible to directly obtain the gain matrix which yields these poles using ES design techniques without solving the filter Ricatti equation. In the single-output, single-input case, this should yield the same result as obtained from solving the Ricatti equation; however, in the multi-input, multi-output case there is not a unique gain matrix

which yields a specified set of poles; thus, ES techniques will necessarily not yield the same results as obtained by the Doyle-Stein procedure. The utility of ES techniques in observer design should be studied in more detail. In addition, the effects on system performance of including flexure modes that were neglected during the design phase needs to be studied.

Acknowledgement

The results on ES design were obtained under Grant NAG-1-217 from NASA Langley Research Center. Mr. William Adams of NASA Langley and Mr. Sanjay Garg of the University of Minnesota were instrumental in developing and interpreting the mathematical models of the ARW-2. Mr. X.G. Li of the University of Minnesota played an important role in calculating the results for the LQ control system. Thanks are also due Dr. Thomas B. Cunningham of Honeywell, Inc. for a number of useful discussions concerning Eigenspace techniques.

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Publications Issued During the Course of this Research

Active Flutter Suppression Using Eigenspace*

and Linear Quadratic Design Techniques

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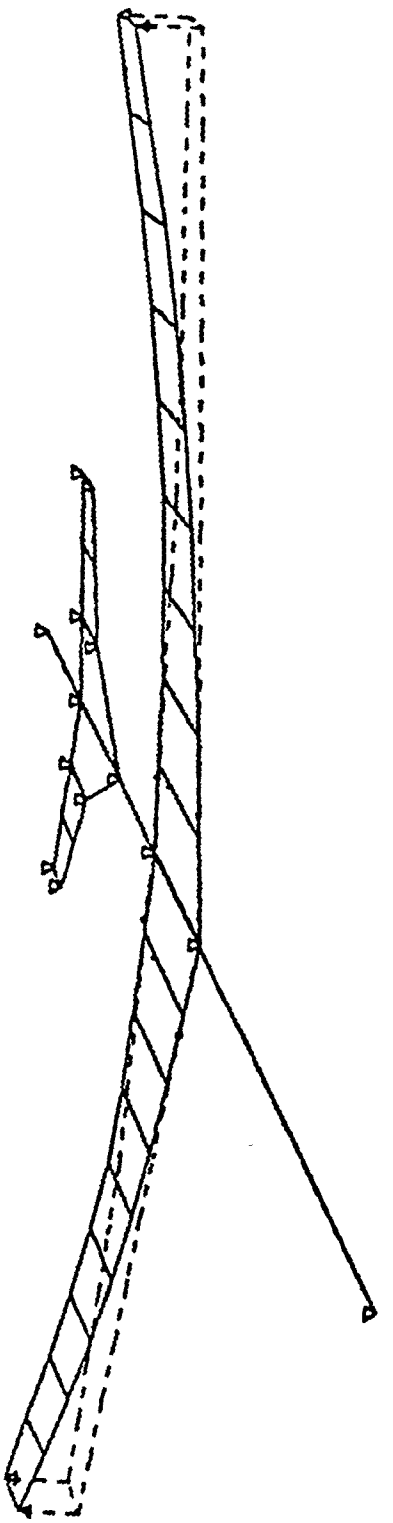
Abstract

Eigenspace (ES) and Linear Quadratic (LQ) techniques are used to design an active flutter suppression system for the DAST ARW-2 flight test vehicle. The performance of the ES and LQ controllers are very similar in meeting control surface activity specifications. The ES controller provides reduced wing root bending moment and shear but torsional stress is slightly higher than with the LQ controller. The ES controller also results in improved flutter boundaries compared with the LQ controller. The LQ controller exhibits significantly better phase margins at the flutter condition than does the ES controller but the LQ design requires large feedback gains on actuator states while the ES does not. This results in reduced overall actuator gain for the LQ design.

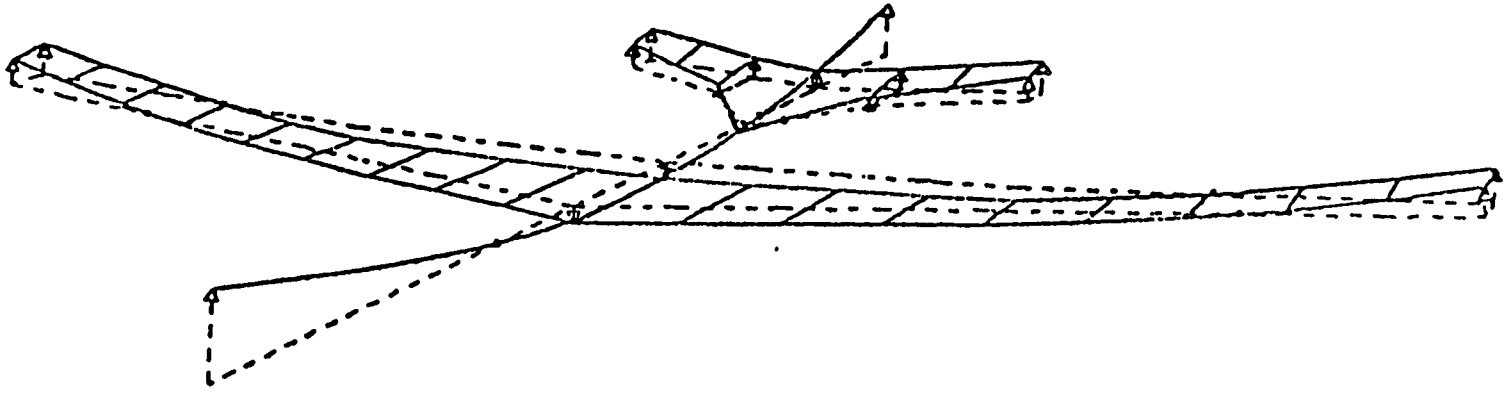
*This paper has been submitted to the AIAA Journal of Guidance, Control, and Dynamics.

Appendix A

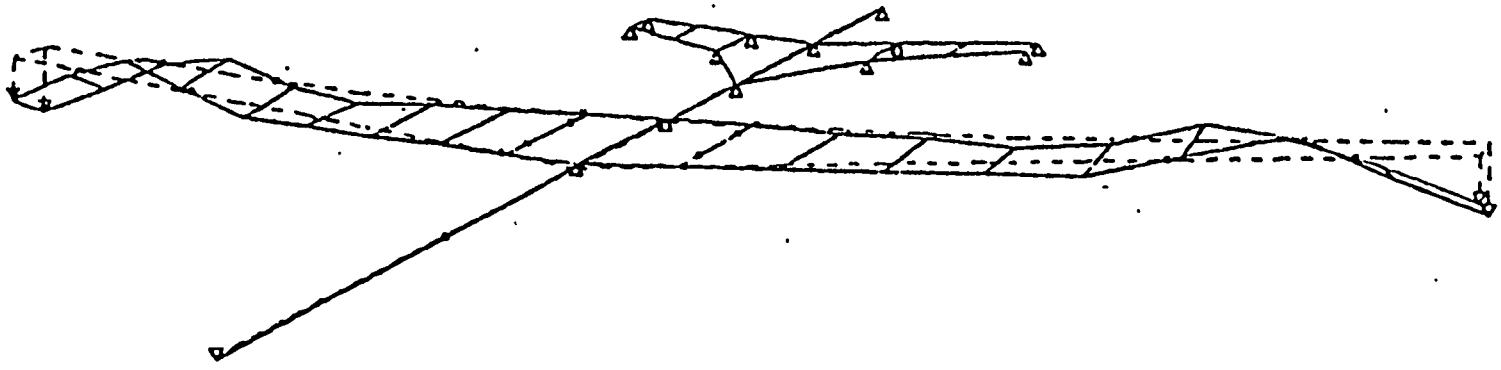
Flexure Mode Shapes



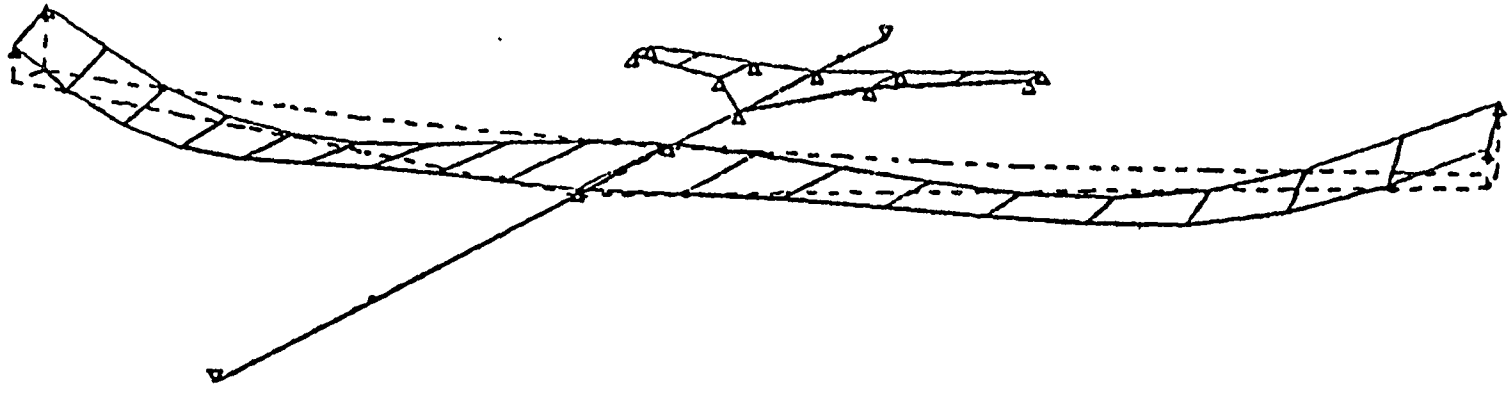
(a) mode 1, frequency = 8.44 hz, generalized mass = 0.0169 $lb\text{-}sec^2/in$



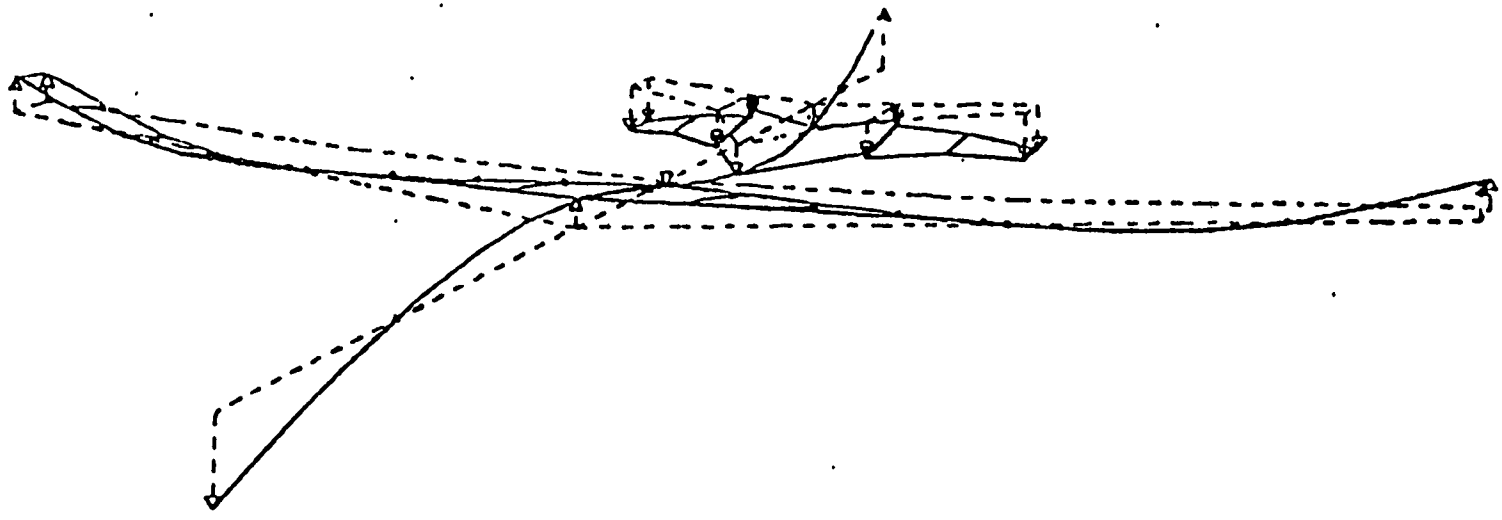
(b) mode 2, frequency = 15.7 hz, generalized mass = 0.326 lb-sec^2/in



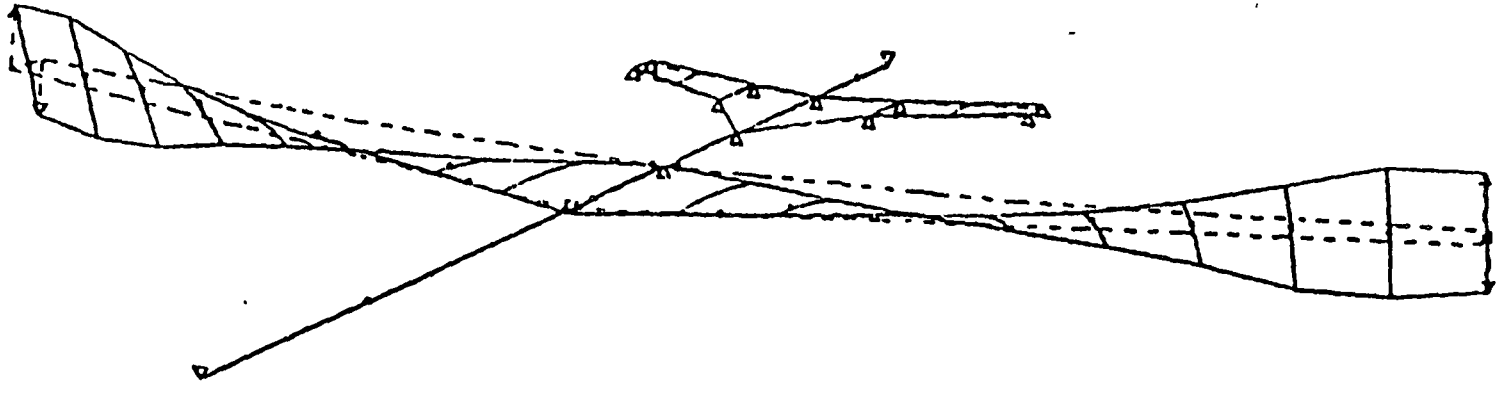
(c) mode 3, frequency = 23.7 hz, generalized mass = 0.0153 lb-sec^2/in



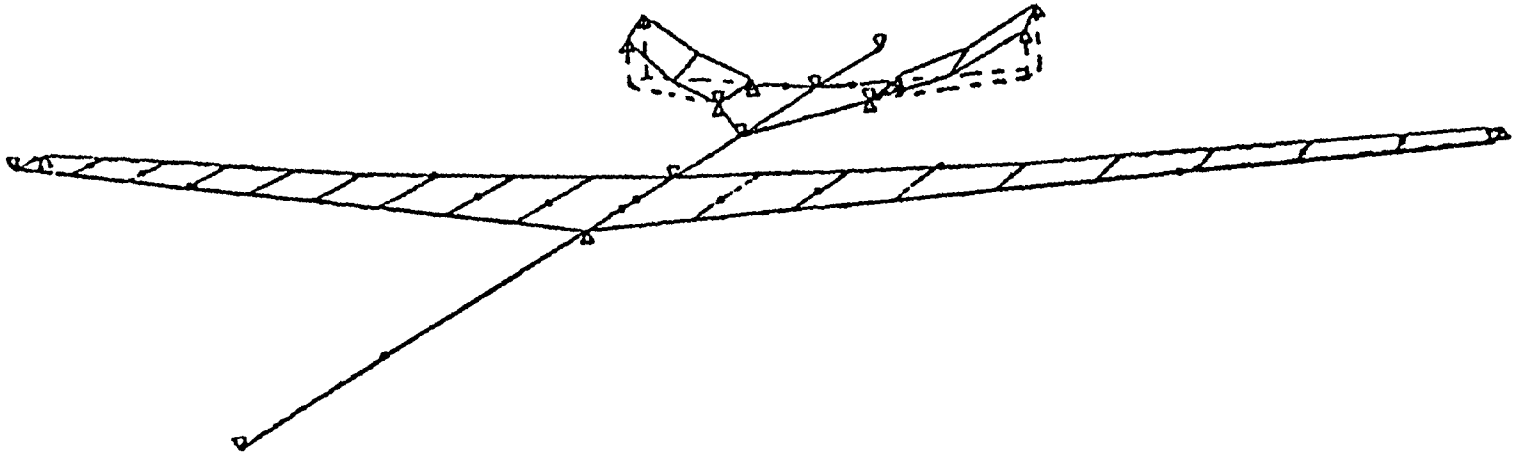
(d) mode 4, frequency = 31.7 hz, generalized mass = 0.0158 lb-sec^2/in



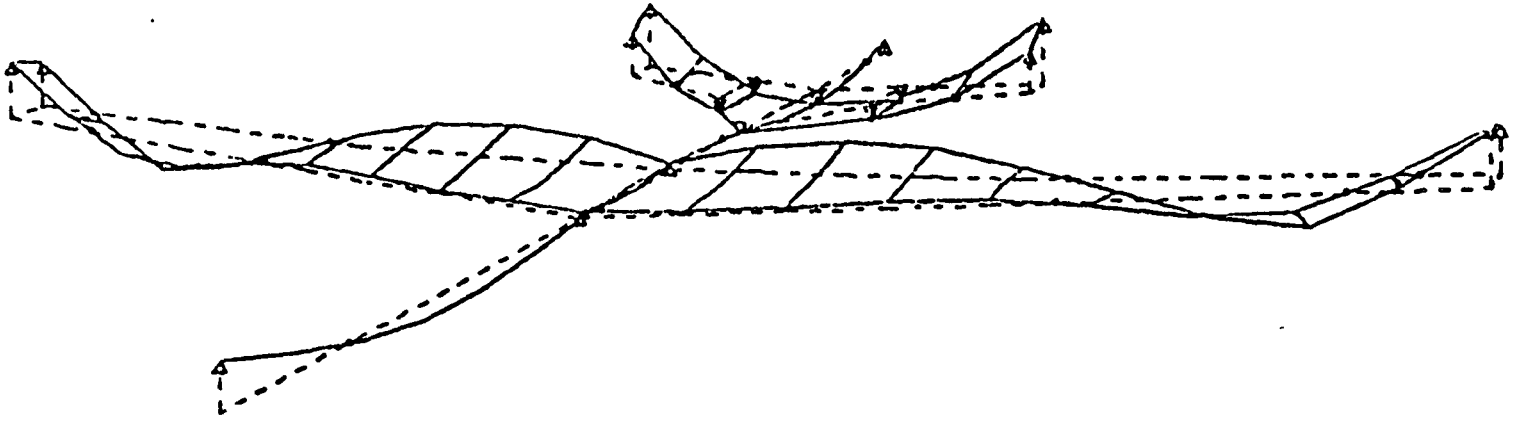
(e) mode 5, frequency = 39.3 hz, generalized mass = 0.178 lb-sec^2/in



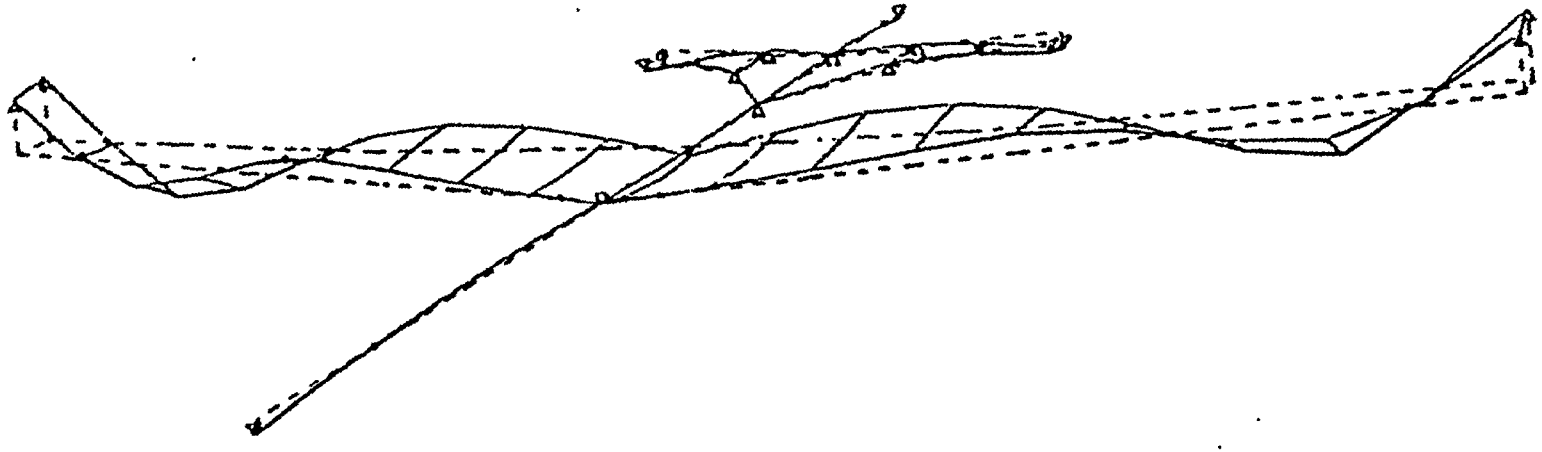
(f) mode 6, frequency = 44.8 hz, generalized mass = 0.593 lb-sec^2/in



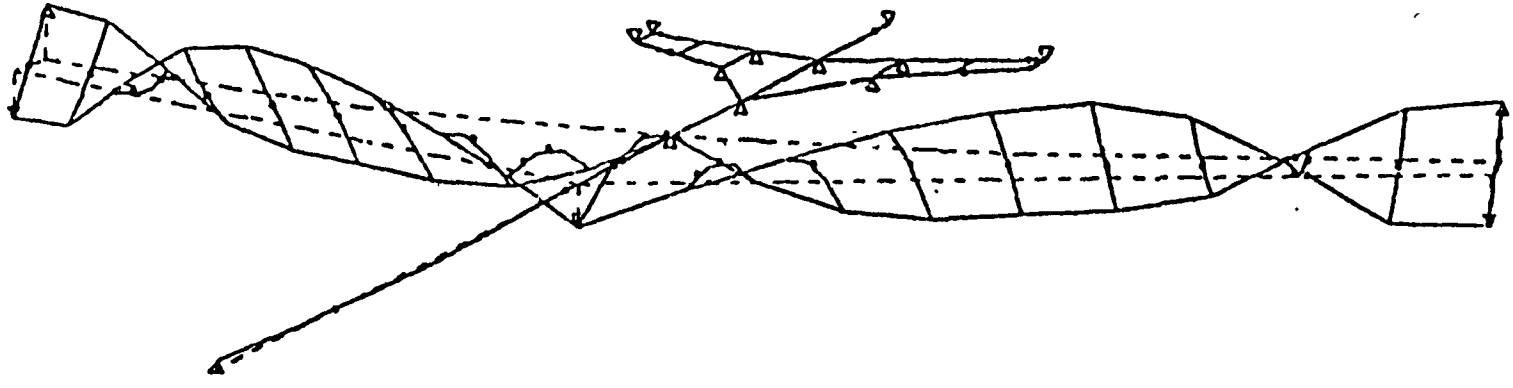
(g) mode 7, frequency = 47.3 hz, generalized mass = 0.00335 lb-sec^2/in



(h) mode 8, frequency = 66.6 hz, generalized mass = 0.0621 lb-sec^2/in



(i) mode 9, frequency = 71.2 hz, generalized mass = 0.0286 lb-sec^2/in



(j) mode 10, frequency = 80.3 hz, generalized mass = 0.696 lb-sec^2/in

Appendix B

Program Descriptions and Listings

FLUTTER

This program generates the coefficient matrices of the first order state equation

$$\dot{x} = Ax + Bu + \Gamma\eta$$

This first order form is derived from the second order form

$$[M_{xx} + \bar{q}\, A_2^x(\frac{\bar{c}}{2V})^2]\ddot{\bar{x}} + [C_s + \bar{q}\, A_1^x(\frac{\bar{c}}{2V})]\dot{\bar{x}} + [K_s + \bar{q}\, A_0^x]\bar{x}$$
$$+ \sum_{i=1}^{L} y_i + [M_{xu} + \bar{q}\, A_2^u(\frac{\bar{c}}{2V})^2]\ddot{\hat{u}} + \bar{q}\, A_1^u(\frac{\bar{c}}{2V})\dot{\hat{u}}$$
$$+ \bar{q}\, A_2^\delta(\frac{\bar{c}}{2V})^2 \frac{\ddot{\delta}}{V} + \bar{q}\, A_1^\delta(\frac{\bar{c}}{2V}) \frac{\dot{\delta}}{V} + \bar{q}\, A_0^\delta \frac{\delta}{V} = 0$$

and i=1,2,...L aerodynamic lag states

$$\dot{y}_i = -I(\frac{2V}{\bar{c}})K_i y_i + D_i^x \dot{\bar{x}} + D_i^u \dot{\hat{u}} + D_i \frac{\dot{\delta}}{V}$$

or

$$M\ddot{\bar{x}} + C\dot{\bar{x}} + K\bar{x} + \sum_{i=1}^{L} y_i + P\ddot{\hat{u}} + Q\dot{\hat{u}} + R\hat{u} + S\ddot{\delta}$$
$$+ T\dot{\delta} + U\delta = 0$$

and

$$\dot{y}_i = -I(\frac{2V}{\bar{c}})K_i y_i + Di\dot{\bar{x}} + Ei\dot{\hat{u}} + Fi\dot{\delta}$$

where

$$M = M_{xx} + \bar{q}\, A_2^x\, (\frac{\bar{c}}{2V})^2$$

$$C = C_s + \bar{q}\, A_1^x\, (\frac{\bar{c}}{2V})$$

$$K = K_s + \bar{q}\, A_0^x$$

$$P = M_{xu} + \bar{q}\, A_2^u \left(\frac{\bar{c}}{2V}\right)^2$$

$$Q = \bar{q}\, A_1^u \left(\frac{\bar{c}}{2V}\right)$$

$$R = \bar{q}\, A_0^u$$

$$S = \bar{q}\, A_2^\delta \left(\frac{\bar{c}}{2V}\right)^2 \left(\frac{1}{V}\right)$$

$$T = \bar{q}\, A_1^\delta \left(\frac{\bar{c}}{2V}\right) \left(\frac{1}{V}\right)$$

$$U = \bar{q}\, A_0^\delta \left(\frac{1}{V}\right)$$

$$Di = D_i^x$$

$$Ei = D_i^u$$

$$Fi = D_i^\delta \left(\frac{1}{V}\right)$$

$\bar{x}$ = modal coordinates and rigid body modes

$\hat{u}$ = control surface deflections

δ = vertical gust velocity

V = forward velocity

$\bar{q} = \frac{1}{2}\rho V^2$

$\bar{c}$ = mean aerodynamic chord

Where the actuator-aileron dynamics are

$$\dot{\hat{u}} = J\bar{u}$$

$$\dot{\bar{u}} = G\bar{u} + Hu$$

where

$$u = \begin{Bmatrix} u_o \\ u_i \end{Bmatrix} = \text{commanded control inputs}$$

$$\hat{u} = \begin{Bmatrix} \bar{u}_o \\ \bar{u}_i \end{Bmatrix} = \text{control surface deflections}$$

$$\bar{u} = \begin{Bmatrix} \bar{u}_o \\ \dot{\bar{u}}_o \\ \ddot{\bar{u}}_o \\ \bar{u}_i \\ \dot{\bar{u}}_i \\ \ddot{\bar{u}}_i \\ \dddot{\bar{u}}_i \end{Bmatrix} = \text{control state}$$

$$G = \begin{bmatrix} 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ -1.774\times10^{7} & -1.438\times10^{5} & -431.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & -1.614\times10^{11} & -5.484\times10^{8} & -1.152\times10^{6} & -933.0 \end{bmatrix}$$

$$J = \begin{bmatrix} .514 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .518 & 0 & 0 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.774\times10^{7} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1.614\times10^{11} \end{bmatrix}$$

And the gust model is

$$\dot{z} = -\left(\frac{V}{\ell}\right)^2 \delta - 2\left(\frac{V}{\ell}\right) z - 2.464\left(\frac{V}{\ell}\right)^2 \eta$$

$$\dot{\delta} = z + 1.732\left(\frac{V}{\ell}\right) \eta$$

where z = an intermediate gust state

ℓ = characteristic gust length

η = white driving noise

Thus, if

$$x = \begin{Bmatrix} \ddot{\bar{x}} \\ \dot{\bar{x}} \\ y_1 \\ y_2 \\ \vdots \\ y_L \\ \ddot{\bar{u}} \\ \delta \\ z \end{Bmatrix}$$

then neglecting $S\ddot{\delta}$ and $P\ddot{\hat{u}}$

$$\Gamma = \begin{Bmatrix} 0 \\ -M^{-1}T \ (1.732)\left(\frac{V}{\ell}\right) \\ F1 \ (1.732)\left(\frac{V}{\ell}\right) \\ F2 \ (1.732)\left(\frac{V}{\ell}\right) \\ \vdots \\ FL \ (1.732)\left(\frac{V}{\ell}\right) \\ 0 \\ (1.732)\left(\frac{V}{\ell}\right) \\ (-2.464)\left(\frac{V}{\ell}\right)^2 \end{Bmatrix}$$

$$
A = \begin{bmatrix}
0 & I & 0 & 0 \cdots & 0 & 0 & 0 & 0 \\
-M^{-1}K & -M^{-1}C & -M^{-1} & -M^{-1} \cdots & -M^{-1} & -M^{-1}(RJ+QJG) & -M^{-1}U & -M^{-1}T \\
0 & D1 & -I\frac{2V}{\bar{c}}K_1 & 0 \cdots & 0 & E1(JG) & 0 & F1 \\
0 & D2 & 0 & -I\frac{2V}{\bar{c}}K_2 \cdots & 0 & E2(JG) & 0 & F2 \\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots \\
0 & DL & 0 & 0 \cdots & -I\frac{2V}{\bar{c}}K_L & EL(JG) & 0 & FL \\
0 & 0 & 0 & 0 \cdots & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 \cdots & 0 & 0 & 0 & 1.0 \\
0 & 0 & 0 & 0 \cdots & 0 & 0 & -(\frac{V}{\ell})^2 & -2(\frac{V}{\ell})
\end{bmatrix}
$$

$$
B = \begin{Bmatrix}
0 \\
-M^{-1}QJH \\
E1\ (JH) \\
E2\ (JH) \\
\vdots \\
EL\ (JH) \\
H \\
0 \\
0
\end{Bmatrix}
$$

The open loop eigenvalues are also output. Open loop eigenvectors can be output as well.

MODAL

This program finds the optimal psuedo-inverse solution to the feedback gains (K) that achieve desired closed loop eigenvalue-eigenvector pairs (see Section on ES theory). The closed loop eigenvalues are also output. Closed loop eigenvectors can be output as well.

COV

This program finds the state covariance matrix for a given driving noise intensity. The rms aileron deflections and rates are determined as well.

LOADS

This program calculates the rms shear, bending moment, and torque at various station along the wing span.

FEEDBKE

This program calculates the closed loop eigenvalues at off design flight conditions.

```
 82/12/15
MNETS     PROGRAM   FLUTTER

00100      PROGRAM FLUTTER(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,ONELE7,
00110+TAPE2=ONELE7,MASS,TAPE1=MASS,TAPE3)
00120C*************************************************************
00130C     THIS PROGRAM INPUTS THE STRUCTURAL MASS,STRUCTURAL
00140C     DAMPING, STRUCTURAL STIFFNESS, AERODYNAMIC INFLUENCE
00150C     MATRICES, FLIGHT CONDITIONS AND OUTPUTS THE A, B, AND W
00160C     MATRICES OF THE LINEAR STATE EQUATION
00170C           DX/DT = AX + BU + WN
00180C     FOR THE INBOARD AND OUTBOARD AILERONS AS CONTROLS.
00190C     OPEN LOOP EIGENVALUES AND EIGENVECTORS CAN BE OUTPUT
00200C     AS WELL.
00210C*************************************************************
00220      REAL MXX(8,8),CS(8,8),KS(8,8),CT(8,8),KT(8,8),CC(8,8),
00230+KK(8,8),A2X(8,8),A1X(8,8),A0X(8,8),MINV(8,8),A2XQ(8,8)
00240+,A1XQ(8,8),A0XQ(8,8),A1(16,16),A2(32,32),QQ(8,2),DUM1(8,7),
00250+DUM2(8,7),DUM3(8,2),DUM4(8,7),DUM5(8,2),DUM6(8,2),DUM7(8,1)
00260      REAL DUM8(8,1),A0U(8,2),A1U(8,2),ET(8,2,4),R(8,2),
00270+ETIG(8,7,4),FI(8,4),A3(40,40),B3(40,2),B2(32,2),II(8,8),
00280+QT(8,8,4),KI(4),A1D(8,1),A0D(8,1),T(8,1),U(8,1),A4(42,42),
00290+B4(42,2),W(42,1),B1(16,2),A5(24,24),B5(24,2)
00300      WRITE(6,8)
00310    8 FORMAT(1X,'INPUT THE ORDER OF THE STRUCTURAL-RIGID BODY STATE VECTO
00320+R(N)')
00330      READ(5,*)N
00340      WRITE(6,10)
00350   10 FORMAT(1X,'INPUT VELOCITY(V), CHORD(C), DYNAMIC PRESSURE(Q), NUMBE
00360+R OF LAG STATES(L)')
00370C
00380C     INPUT FLIGHT CONDITIONS
00390C
00400      READ(5,*)V,C,Q,L
00410      DO 20 I=1,L
00420      WRITE(6,25)I
00430C
00440C     INPUT AERODYNAMIC LAG FREQUENCIES
00450C
00460      READ(5,*)KI(I)
00470   20 CONTINUE
00480   25 FORMAT(1X,'INPUT KI(',I1,')')
00490      N2=2*N
00500      KEND=N*(L+2)
00510      KEND0=KEND+7
00520      KEND2=KEND0+2
00530      K5=2*N+7
```

```
00560+,DUM5,DUM6,DUM7,DUM8,A0U,A1U,ET,R,ETJG,TT,A3,B3,B2,FT,DT,KT,A1D,
00570+A0D,T,U,A4,B1,B4,W,A5,B5)
00580      STOP
00590      END
00600C
00610C
00620      SUBROUTINE PROG1(N,N2,KEND,KEND0,KEND2,K5,V,C,Q,L,MXX,CS,KS,CT,KT,
00630+CC,KK,A2X,A1X,A0X,MINV,A2XQ,A1XQ,A0XQ,A1,A2,QQ,DUM1,DUM2,DUM3,DUM4
00640+,DUM5,DUM6,DUM7,DUM8,A0U,A1U,ET,R,ETJG,II,A3,B3,B2,FT,DI,KI,A1D,
00650+A0D,T,U,A4,B1,B4,W,A5,B5)
00660      REAL LAMDA,LL
00670      INTEGER FLAG
00680      COMMON KRET(42)
00690      REAL MXX(N,N),CS(N,N),KS(N,N),CT(N,N),KT(N,N),CC(N,N),KK(N,N),
00700+A2X(N,N),A1X(N,N),A0X(N,N),MINV(N,N),A2XQ(N,N),A1XQ(N,N),A0XQ(N,N)
00710+,A1(N2,N2),A2(KEND,KEND),QQ(N,2),DUM1(N,7),DUM2(N,7),DUM3(N,2),
00720+DUM4(N,7),DUM5(N,2),DUM6(N,2),DUM7(N,1),DUM8(N,1),A0U(N,2)
00730      REAL A1U(N,2),
00740+EI(N,2,L),G(7,7),H(7,2),R(N,2),JG(2,7),EIJG(N,7,L),FI(N,L),
00750+JH(2,2),A3(KEND0,KEND0),B3(KEND0,2),B2(KEND,2),II(N,N),DI(N,N,L)
00760      REAL KI(L),A1D(N,1),A0D(N,1),T(N,1),U(N,1),A4(KEND2,KEND2),
00770+B1(N2,2),B4(KEND2,2),W(KEND2,1),
00780+A5(K5,K5),B5(K5,2),JJ(2,7)
00790      REAL AA(42,42),BB(42,2)
00800      CALL GETPF(5HTAPE1,4HMASS,0,0)
00810C
00820C     READ IN STRUCTURAL DATA
00830C
00840    2 FORMAT(E16.8)
00850      DO 1 I=1,N
00860      DO 1 J=1,N
00870      READ(1,101)MXX(I,J),CS(I,J),KS(I,J)
00880    1 CONTINUE
00890      DO 3 I=1,7
00900      DO 3 J=1,7
00910C
00920C     READ IN ACTUATOR DATA
00930C
00940      READ(1,101)G(I,J)
00950    3 CONTINUE
00960      DO 4 I=1,7
00970      READ(1,101)(H(I,J),J=1,2)
00980    4 CONTINUE
00990      DO 5 J=1,7
01000      READ(1,101)(JJ(I,J),I=1,2)
01010    5 CONTINUE
01020      CALL GETPF(5HTAPE2,6HONELE7,0,0)
01030C
01040C     READ IN AERODYNAMIC INFLUENCE DATA
```

```
01060      DO 430 K=1,1
01070      DO 410 J=1,N
01080      DO 410 I=1,N
01090      READ(2,109)A0X(I,J),A1X(I,J),A2X(I,J),QI(I,J,K)
01100  410 CONTINUE
01110      DO 420 J=1,2
01120      DO 420 I=1,N
01130      READ(2,101)A0U(I,J),A1U(I,J),EI(I,J,K)
01140  420 CONTINUE
01150      DO 430 I=1,N
01160      READ(2,101)A0D(I,K),A1D(I,K),FI(I,K)
01170      FI(I,K)=FI(I,K)/V
01180  430 CONTINUE
01190  101 FORMAT(3E16.8)
01200  109 FORMAT(4E16.8)
01210      EPS=1.E-10
01220      TEMP=0.5*(C/2./V)**2
01230      TEMP2=0.5*C/2./V
01240      CALL SCAMAT(TEMP,A2X,A2X0,N,N)
01250      CALL MATADD(MXX,A2X0,MINV,N,N)
01260C
01270C     FIND INVERSE(MINV)
01280C
01290      CALL MXLNEQ(MINV,N,N,DET,JRANK,EPS,A2X0,0)
01300      CALL SCAMAT(TEMP2,A1X,A1X0,N,N)
01310      CALL MATADD(CS,A1X0,CT,N,N)
01320      CALL SCAMAT(Q,A0X,A0X0,N,N)
01330      CALL MATADD(KS,A0X0,KT,N,N)
01340      CALL SCAMAT(-1.,MINV,MINV,N,N)
01350      CALL MATMUL(MINV,KT,KK,N,N,N)
01360      CALL MATMUL(MINV,CT,CC,N,N,N)
01370      DO 15 I=1,N
01380      DO 14 J=1,N
01390      II(I,J)=0.
01400   14 CONTINUE
01410      II(I,I)=1.0
01420   15 CONTINUE
01430      N21=N2+1
01440      DO 18 I=1,N
01450      DO 18 J=1,N
01460      A1(I,J)=0.
01470      A1(I,J+N)=II(I,J)
01480      A1(I+N,J)=KK(I,J)
01490      A1(I+N,J+N)=CC(I,J)
01500   18 CONTINUE
01510      IF(L.EQ.0)GO TO 100
01520      DO 20 I=1,KEND
01530      DO 20 J=1,KEND
01540      A2(I,J)=0.
01550   20 CONTINUE
```

```
      A2(I,J)=A1(I,J)
      A2(I+N,J)=A1(I+N,J)
      A2(I,J+N)=A1(I,J+N)
      A2(I+N,J+N)=A1(I+N,J+N)
      DO 25 K=1,L
      KN=K+N
      A2(I+N,KN+N+J)=MINV(I,J)
      A2(KN+N+I,J+N)=DI(I,J,K)
      A2(KN+N+I,KN+N+I)=-2.*V*KT(K)/C
   25 CONTINUE
      WRITE(6,27)
   27 FORMAT(1X,'SHOULD THE CONTROL MATRIX B ALSO BE CALCULATED? YES(1),
     + NO(0)')
      READ(5,28)MM
   28 FORMAT(I1)
      IF(MM.EQ.1)GO TO 30
      FLAG=2
      GO TO 200
   30 SCA=Q+C/2./V
      CALL SCAMAT(SCA,A1U,QQ,N,2)
      CALL SCAMAT(Q,A0U,R,N,2)
      DO 36 I=1,KENDD
      DO 36 J=1,KENDD
      A3(I,J)=0.
   36 CONTINUE
      DO 37 I=1,KEND
      DO 37 J=1,KEND
      A3(I,J)=A2(I,J)
   37 CONTINUE
      DO 38 I=1,7
      DO 38 J=1,7
      A3(KEND+I,KEND+J)=G(I,J)
   38 CONTINUE
      CALL MATMUL(JJ,G,JG,2,7,7)
      CALL MATMUL(QQ,JG,DUM1,N,2,7)
      CALL MATMUL(R,JJ,DUM2,N,2,7)
      CALL MATADD(DUM2,DUM1,DUM2,N,7)
      CALL MATMUL(MINV,DUM2,DUM1,N,N,7)
      DO 40 K=1,L
      DO 39 I=1,N
      DO 39 J=1,2
      DUM3(I,J)=EI(I,J,K)
   39 CONTINUE
      CALL MATMUL(DUM3,JG,DUM4,N,2,7)
      DO 40 I=1,N
      DO 40 J=1,7
      ETJG(I,J,K)=DUM4(I,J)
   40 CONTINUE
      DO 45 I=1,N
```

```
02080      A3(KN+N+I,J+KEND)=EIJG(I,J,K)
02090      DO 45 K=1,L
02100      KN=K*N
02110      A3(KN+N+I,J+KEND)=EIJG(I,J,K)
02120   45 CONTINUE
02130      CALL MATMUL(JJ,H,JH,2,2,2)
02140      CALL MATMUL(QQ,JH,DUM3,N,2,2)
02150      CALL MATMUL(MINV,DUM3,DUM5,N,N,2)
02160      DO 47 K=1,L
02170      DO 46 I=1,N
02180      DO 46 J=1,2
02190      DUM3(I,J)=EI(I,J,K)
02200   46 CONTINUE
02210      CALL MATMUL(DUM3,JH,DUM6,N,2,2)
02220      DO 47 I=1,N
02230      DO 47 J=1,2
02240      EIJG(I,J,K)=DUM6(I,J)
02250   47 CONTINUE
02260      DO 48 I=1,N
02270      DO 48 J=1,2
02280      B3(I,J)=0.
02290      B3(I+N,J)=DUM5(I,J)
02300      DO 48 K=1,L
02310      KN=K*N
02320      B3(I+KN+N,J)=EIJG(I,J,K)
02330   48 CONTINUE
02340      DO 49 I=1,7
02350      DO 49 J=1,2
02360      B3(I+KEND,J)=H(I,J)
02370   49 CONTINUE
02380      WRITE(6,50)
02390   50 FORMAT(1X,'SHOULD THE NOISE VECTOR (W) ALSO BE CALCULATED? YES(1),
02400+NO(0)'/)
02410      READ(5,28)MM
02420      IF(MM.EQ.1)GO TO 55
02430      FLAG=3
02440      GO TO 200
02450   55 WRITE(6,56)
02460   56 FORMAT(1X,'INPUT CHARACTERISTIC LENGTH(LL)'/)
02470      READ(5,*)LL
02480      TEMP3=TEMP2/V
02490      CALL SCAMAT(TEMP3,AID,T,N,1)
02500      Q1=Q/V
02510      CALL SCAMAT(Q1,AOD,U,N,1)
02520      CALL MATMUL(MINV,U,DUM7,N,N,1)
02530      CALL MATMUL(MINV,T,DUM8,N,N,1)
02540      DO 58 I=1,KENDD
02550      DO 58 J=1,2
02560      B4(I,J)=B3(I,J)
02570      B4(KENDD+1,J)=0.
```

```
      DO 59 I=1,KEND2
      DO 59 J=1,KEND2
      A4(I,J)=0.
   59 CONTINUE
      DO 60 I=1,KENDD
      DO 60 J=1,KENDD
      A4(I,J)=A3(I,J)
   60 CONTINUE
      A4(KENDD+1,KENDD+2)=1.
      A4(KENDD+2,KENDD+1)=-1.001*(V/LL)**2
      A4(KENDD+2,KENDD+2)=-2.001*V/LL
      DO 62 I=1,N
      A4(I+N,KENDD+1)=DUM7(I,1)
      A4(I+N,KENDD+2)=DUM8(I,1)
      DO 62 K=1,L
      KN=K*N
      A4(I+KN+N,KENDD+2)=FI(I,K)
   62 CONTINUE
      DO 64 I=1,KEND2
      W(I,1)=0.
   64 CONTINUE
      XX=1.732*V/LL
      W(KENDD+1,1)=XX
      W(KENDD+2,1)=-2.464*(V/LL)**2
      CALL SCAMAT(XX,DUM8,DUM8,N,1)
      DO 66 I=1,N
      W(I+N,1)=DUM8(I,1)
      DO 66 K=1,L
      KN=K*N
      W(I+N+KN,1)=XX*FI(I,K)
   66 CONTINUE
      FLAG=4
      GO TO 200
  100 WRITE(6,27)
      READ(5,28)MM
      IF(MM.EQ.1)GO TO 102
      FLAG=1
      GO TO 200
  102 SCA=0*C/2./V
      CALL SCAMAT(SCA,A10,00,N,2)
      CALL SCAMAT(0,A0U,R,N,2)
      DO 106 I=1,K5
      DO 106 J=1,K5
      A5(I,J)=0.
  106 CONTINUE
      CALL MATMUL(JJ,H,JH,2,7,2)
      CALL MATMUL(00,JH,DUM3,N,2,2)
      CALL MATMUL(MINV,DUM3,DUM5,N,N,2)
      DO 108 I=1,N
```

```
03110      B5(I+N,J)=DUM5(I,J)
03120  108 CONTINUE
03130      DO 110 I=1,7
03140      DO 110 J=1,2
03150      B5(I+N2,J)=H(I,J)
03160  110 CONTINUE
03170      DO 112 I=1,N2
03180      DO 112 J=1,N2
03190      A5(I,J)=A1(I,J)
03200  112 CONTINUE
03210      CALL MATMUL(JJ,G,JG,2,7,7)
03220      CALL MATMUL(QQ,JG,DUM1,N,2,7)
03230      CALL MATMUL(R,JJ,DUM2,N,2,7)
03240      CALL MATADD(DUM2,DUM1,DUM2,N,7)
03250      CALL MATMUL(MINV,DUM2,DUM1,N,N,7)
03260      DO 114 I=1,N
03270      DO 114 J=1,7
03280      A5(I+N,J+N2)=DUM1(I,J)
03290  114 CONTINUE
03300      DO 116 I=1,7
03310      DO 116 J=1,7
03320      A5(I+N2,J+N2)=G(I,J)
03330  116 CONTINUE
03340      FLAG=5
03350  200 WRITE(6,201)
03360  201 FORMAT(1X,'INPUT RETAIN(1) OR DELETE(0) FOR EACH STATE MEMBER'/)
03370      KSTATE=N
03380      KSUM=0
03390      READ(5,*)(KRET(J),J=1,KSTATE)
03400      DO 205 I=1,KSTATE
03410      KRET(I+N)=KRET(I)
03420      IF(KRET(I).EQ.1)KSUM=KSUM+1
03430  205 CONTINUE
03440      IF(L.EQ.0)GO TO 207
03450      DO 206 K=1,L
03460      DO 206 I=1,N
03470      KN=N*(K+1)+I
03480      KRET(KN)=KRET(I)
03490  206 CONTINUE
03500      DO 209 I=1,9
03510      KRET(KEND+I)=I
03520  209 CONTINUE
03530  207 GO TO(310,320,330,340,350),FLAG
03540  310 KSUM=2*KSUM
03550      CALL PROG2(FLAG,KSUM,A1,B1,N2,AA,BB)
03560      GO TO 900
03570  320 KSUM=(L+2)*KSUM
03580      CALL PROG2(FLAG,KSUM,A2,B2,KEND,AA,BB)
03590      GO TO 900
```

```
      GO TO 900
  340 KSUM=9+(I+2)*KSUM
      DO 341 I=1,KEND2
      IF(KRET(I).EQ.0)GO TO 341
      WRITE(3,2)W(I,1)
  341 CONTINUE
      CALL PROG2(FLAG,KSUM,A4,B4,KEND2,AA,BB)
      GO TO 900
  350 KSUM=7+2*KSUM
      DO 351 I=N21,K5
      KRET(I)=1
  351 CONTINUE
      CALL PROG2(FLAG,KSUM,A5,B5,K5,AA,BB)
  900 RETURN
      END
C
C
      SUBROUTINE MATDEL(A,N,AA,KSUM)
C
C     THIS ROUTINE DELETES DESIRED STATES FROM THE A MATRIX
C
      DIMENSION A(N,N),AA(KSUM,KSUM),JRET(40)
      COMMON KRET(42)
      J=0
      DO 100 I=1,N
      IF(KRET(I).EQ.0)GO TO 100
      J=J+1
      JRET(J)=I
  100 CONTINUE
      DO 200 I=1,KSUM
      DO 200 J=1,KSUM
      AA(I,J)=A(JRET(I),JRET(J))
  200 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE MATDELB(B,N,BB,KSUM)
C
C     THIS ROUTINE DELETES DESIRED STATES FROM THE B MATRIX
C
      DIMENSION B(N,2),BB(KSUM,2),JRET(40)
      COMMON KRET(42)
      J=0
      DO 100 I=1,N
      IF(KRET(I).EQ.0)GO TO 100
      J=J+1
      JRET(J)=I
  100 CONTINUE
```

"PAGE MISSING FROM AVAILABLE VERSION"

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04130       RR(I,J)=R(IRET(I),J)
04140   200 CONTINUE
04150       RETURN
04160       END
04170C
04180C
04190C.....T(M,P)=A(M,N) * U(N,P)
04200       SUBROUTINE MATMUL(A,U,T,M,N,P)
04210       INTEGER P
04220       DIMENSION A(M,N),U(N,P),T(M,P)
04230       DO 1 I=1,M
04240       DO 1 J=1,P
04250       T(I,J)=0.
04260     1 CONTINUE
04270       DO 2 I=1,M
04280       DO 2 J=1,P
04290       DO 2 K=1,N
04300       T(I,J)=A(I,K)*U(K,J)+T(I,J)
04310     2 CONTINUE
04320       RETURN
04330       END
04340C
04350C
04360C.....C(M,N)=A(M,N) + B(M,N)
04370       SUBROUTINE MATADD(A,B,C,M,N)
04380       DIMENSION A(M,N),B(M,N),C(M,N)
04390       DO 3 I=1,M
04400       DO 3 J=1,N
04410       C(I,J)=A(I,J)+B(I,J)
04420     3 CONTINUE
04430       RETURN
04440       END
04450C
04460C
04470C.....B(M,N)=S * A(M,N)
04480       SUBROUTINE SCAMAT(S,A,B,M,N)
04490       DIMENSION A(M,N),B(M,N)
04500       DO 8 I=1,M
04510       DO 8 J=1,N
04520       B(I,J)=S*A(I,J)
04530     8 CONTINUE
04540       RETURN
04550       END
04560C
04570C
04580C.....V(N,M)= A(M,N) TRANSPOSED
04590       SUBROUTINE MATTRA(A,V,M,N)
04600       DIMENSION A(M,N),V(N,M)
04610       DO 14 I=1,M
```

```
04630          V(J,I)=R(I,J)
04640    14 CONTINUE
04650       RETURN
04660       END
04670C
04680C
04690       SUBROUTINE PROG2(FLAG,KSUM,A,B,IP,AA,BB)
04700       INTEGER FLAG,IV1(42)
04710       REAL AA(KSUM,KSUM),BB(KSUM,2),A(IP,IP),B(IP,2),FV1(42)
04720       REAL WR(42),WI(42),Z(42,42)
04730       GO TO(310,320,330,340,350),FLAG
04740   100 FORMAT(3E16.8)
04750   310 CALL MATDEL(A,IP,AA,KSUM)
04760       DO 311 I=1,KSUM
04770       DO 311 J=1,KSUM
04780       WRITE(3,100)AA(I,J)
04790   311 CONTINUE
04800       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1)
04810       GO TO 900
04820   320 CALL MATDEL(A,IP,AA,KSUM)
04840       DO 321 J=1,KSUM
04850       WRITE(3,100)AA(I,J)
04860   321 CONTINUE
04870       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1)
04880       GO TO 900
04890   330 CALL MATDEL(A,IP,AA,KSUM)
04900       CALL MATDELB(B,IP,BB,KSUM)
04910       DO 331 I=1,KSUM
04920       DO 331 J=1,KSUM
04930       WRITE(3,100)AA(I,J)
04940   331 CONTINUE
04950       DO 332 I=1,KSUM
04960       WRITE(3,100)BB(I,1),BB(I,2)
04970   332 CONTINUE
04980       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1)
04990       GO TO 900
05000   340 CALL MATDEL(A,IP,AA,KSUM)
05010       CALL MATDELB(B,IP,BB,KSUM)
05020       DO 341 I=1,KSUM
05030       DO 341 J=1,KSUM
05040       WRITE(3,100)AA(I,J)
05050   341 CONTINUE
05060       DO 342 I=1,KSUM
05070       WRITE(3,100)BB(I,1),BB(I,2)
05080   342 CONTINUE
05090       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1)
05100       GO TO 900
05110   350 CALL MATDEL(A,IP,AA,KSUM)
05120       CALL MATDELB(B,IP,BB,KSUM)
05130       DO 351 I=1,KSUM
05140       DO 351 J=1,KSUM
```

```
05160   351 CONTINUE
05170       DO 352 I=1,KSUM
05180       WRITE(3,100)BB(I,1),BB(I,2)
05190   352 CONTINUE
05200       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1)
05210   900 CALL REPLACE(5HTAPE3,5HFILE9,0,0)
05220       RETURN
05230       END
05240C
05250C
05260       SUBROUTINE PROG3(N,AA,WR,WI,Z,IV1,FV1)
05270       COMMON KRET(42)
05280       DIMENSION AA(N,N),WR(N),WI(N),Z(N,N),IV1(N),FV1(N)
05290C
05300C      CALCULATE EIGENVALUES AND EIGENVECTORS OF AA
05310C
05320       CALL RG(N,N,AA,WR,WI,1,Z,IV1,FV1,IERR)
05330       WRITE(6,23)
05340    23 FORMAT(41X,'OPEN LOOP EIGENVALUES'/31X,'REAL PART',20X,
05350+'IMAGINARY PART')
05360       DO 30 I=1,N
05370       WRITE(6,25)I,WR(I),WI(I)
05380    25 FORMAT(20X,I2,'.',2X,E15.4,15X,E15.4)
05390    30 CONTINUE
05400       WRITE(6,55)
05410    55 FORMAT(1X,'DO YOU WANT THE OPEN LOOP EIGENVECTORS OUTPUT? YES(1)
05420+,NO(0)')
05430       READ(5,*)MM
05440       IF(MM.EQ.0)GO TO 60
05450       WRITE(6,64)
05460    64 FORMAT(1X,'INPUT PRINT(1) OR SUPPRESS(0) FOR EACH EIGENVECTOR')
05470       READ(5,*)(KRET(J),J=1,N)
05480       WRITE(6,34)
05490    34 FORMAT(41X,'OPEN LOOP EIGENVECTORS'/31X,'REAL PART',20X,
05500+'IMAGINARY PART')
05510       IFLAG=0
05520       DO 50 J=1,N
05530       IF(KRET(J).EQ.0)GO TO 49
05540       IF(WI(J).NE.0.0)GO TO 35
05550       DO 37 I=1,N
05560       IF(I.NE.1)GO TO 36
05570       WRITE(6,38)J,Z(1,J)
05580    38 FORMAT(//20X,I2,'.',2X,E15.4,20X,'0.0')
05590       GO TO 37
05600    36 WRITE(6,39)Z(I,J)
05610    39 FORMAT(25X,E15.4,20X,'0.0 )
05620    37 CONTINUE
05630       GO TO 50
05640    35 IF(IFLAG.EQ.1)GO TO 40
05650       IFLAG=1
```

```
05680       WRITE(6,43)J,Z(I,J),Z(I,J+1)
05690    43 FORMAT(//20X,I2,'.',2X,E15.4,15X,E15.4)
05700       GO TO 41
05710    42 WRITE(6,44)Z(I,J),Z(I,J+1)
05720    44 FORMAT(25X,E15.4,15X,E15.4)
05730    41 CONTINUE
05740       GO TO 50
05750    40 J1=J-1
05760       WRITE(6,45)J,J1
05770    45 FORMAT(//20X,I2, '.',5X,'CONJUGATE OF EIGENVECTOR',1X,I2,'.')
05780    49 IFLAG=0
05790    50 CONTINUE
05800    60 RETURN
05810       END
```

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82/12/15
MNETS     PROGRAM   MODAL

00100      PROGRAM MODAL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,FILE3,
00110+TAPE1=FILE3,TAPE3)
00120C*****************************************************************
00130C     THIS PROGRAM INPUTS THE STATE MATRICES(OUTPUT
00140C     OF FLUTTER) A, B, W, DESIRED CLOSED
00150C     LOOP EIGENVALUES AND EIGENVECTORS, THE EIGENVECTOR
00160C     WEIGHTING MATRIX AND OUTPUTS THE GAIN MATRIX K WHERE
00170C                    U = KX
00180C     THE CLOSED LOOP EIGENVALUES AND EIGENVECTORS THAT WERE
00190C     ACHIEVED CAN BE OUTPUT AS WELL.
00200C*****************************************************************
00210      DIMENSION A(42,42),WR(2,42),WI(2,42),Z(42,42),IV1(42),
00220+FV1(42),VECRD(42,42),VECID(42,42),ERD(42),EID(42),KRET(2,42),W(42,1),
00230+B(42,2)
00240      WRITE(6,10)
00250   10 FORMAT(1X,'INPUT ORDER OF SYSTEM')
00260      READ(5,*)N
00270      CALL PROG1(N,A,WR,WI,Z,IV1,FV1,VECRD,VECID,ERD,EID,KRET,
00280+W,B)
00290      STOP
00300      END
00310C
00320C
00330      SUBROUTINE PROG1(N,A,WR,WI,Z,IV1,FV1,VECRD,VECID,ERD,EID,
00340+KRET,W,B)
00350      DIMENSION A(N,N),WR(2,N),WI(2,N),Z(N,N),IV1(N),FV1(N),
00360+VECRD(N,N),VECID(N,N),ERD(N),EID(N),KRET(2,N),W(N,1),B(N,2)
00370      REAL II(42,42),LB(84,4),LBT(4,84)
00380      DIMENSION QB(84,84),VB(84,1),YB(84,84),PP(42,2),NV(42,1),
00390+BR(84,4),C(42,42),P(84,1)
00400      N2=N*2
00410      NM1=N-1
00420      NM2=N-2
00430      WRITE(6,11)
00440   11 FORMAT(1X,'IS THE NOISE VECTOR (W) BEING INPUT? YES(1), NO(0)')
00450      READ(5,12)MM
00460   12 FORMAT(I1)
00470C
00480C     READ IN STATE MATRICES OUTPUT FROM PROGRAM: FLUTTER
00490C
00500      CALL GETPF(5HTAPE1,5HFILE3,0,0)
00510      IF(MM.EQ.0)GO TO 14
00520      DO 13 I=1,N
00530      READ(1,15)W(I,1)
```

```
00560   14 DO 16 I=1,N
00570      DO 16 J=1,N
00580      READ(1,15)A(I,J)
00590   16 CONTINUE
00600      DO 17 I=1,N
00610      READ(1,15)B(I,1),B(I,2)
00620   17 CONTINUE
00630      DO 24 I=1,N
00640      DO 24 J=1,N
00650      VECRD(I,J)=A(I,J)
00660   24 CONTINUE
00670C
00680C     CALCULATE OPEN LOOP EIGENVALUES AND EIGENVECTORS
00690C
00700      CALL RG(N,N,VECRD,ERD,EID,1,Z,IV1,FV1,IERR)
00710      WRITE(6,23)
00720   23 FORMAT(41X,'OPEN LOOP EIGENVALUES'/31X,'REAL PART',20X,
00730+'IMAGINARY PART')
00740      DO 30 I=1,N
00750      WRITE(6,25)I,ERD(I),EID(I)
00760   25 FORMAT(20X,I2,'.',2X,E15.4,15X,E15.4)
00770   30 CONTINUE
00780      IFLAG=0
00790      DO 50 J=1,N
00800      IF(EID(J).NE.0.0)GO TO 35
00810      KRET(1,J)=0
00820      DO 37 I=1,N
00830      IF(I.NE.1)GO TO 36
00840      VECRD(1,J)=Z(1,J)
00850      VECID(1,J)=0.0
00860      GO TO 37
00870   36 VECRD(I,J)=Z(I,J)
00880      VECID(I,J)=0.0
00890   37 CONTINUE
00900      GO TO 50
00910   35 KRET(1,J)=IFLAG
00920      IF(IFLAG.EQ.1)GO TO 40
00930      IFLAG=1
00940      DO 41 I=1,N
00950      IF(I.NE.1)GO TO 42
00960      VECRD(1,J)=Z(1,J)
00970      VECID(1,J)=Z(1,J+1)
00980      GO TO 41
00990   42 VECRD(I,J)=Z(I,J)
01000      VECID(I,J)=Z(I,J+1)
01010   41 CONTINUE
01020      GO TO 50
01030   40 J1=J-1
01040      DO 44 I=1,N
```

```
01060      VECRD(I,I1)=-VECRD(I,I)
01070    4A CONTINUE
01080       IFLAG=0
01090    50 CONTINUE
01100       WRITE(6,100)
01110   100 FORMAT(1X,'INPUT RETAIN(1) OR CHANGE(0) FOR EACH EIGENSOLUTION ')
01120       READ(5,*)(KRET(2,J),J=1,N)
01130       DO 110 I=1,N
01140       IF(KRET(2,I).EQ.1)GO TO 120
01150       IF(KRET(1,I).EQ.0)GO TO 105
01160       I1=I-1
01170       WRITE(6,107)I,I1
01180   107 FORMAT(1X,'DESIRED EIGENVALUE ',I2,'. WILL BE ENTERED AS THE CONJ
01190+UGATE OF DESIRED EIGENVALUE ',I2,'.')
01200       ERD(I)=ERD(I1)
01210       EID(I)=-EID(I1)
01220       GO TO 110
01230   105 IF(VECRD(NM1,I).NE.0..OR.VECRD(N,I).NE.0.)GO TO 110
01240       WRITE(6,101)I
01250   101 FORMAT(1X,'INPUT DESIRED/REAL PART, IMAGINARY PART OF NEW',1X,
01260+'EIGENVALUE ',I2,'.'/)
01270       READ(5,*)ERD(I),EID(I)
01280       GO TO 110
01290   120 DO 121 J=1,2
01300       WR(J,I)=0.
01310       WI(J,I)=0.
01320   121 CONTINUE
01330   110 CONTINUE
01340       DO 140 I=1,N
01350       IF(KRET(2,I).EQ.1)GO TO 140
01360       IF(KRET(1,I).EQ.0)GO TO 113
01370       I1=I-1
01380       WRITE(6,114)I,I1
01390   114 FORMAT(1X,'DESIRED EIGENVECTOR ',I2,'. WILL BE ENTERED AS THE CONJ
01400+UGATE OF DESIRED EIGENVECTOR ',I2,'.')
01410       DO 115 J=1,N
01420       VECRD(J,I)=VECRD(J,I1)
01430       VECID(J,I)=-VECID(J,I1)
01440   115 CONTINUE
01450       GO TO 140
01460   113 WRITE(6,111)I
01470   111 FORMAT(1X,'INPUT DESIRED REAL PART, IMAGINARY PART OF NEW EIGENVECT
01480+OR ',I2,'.')
01490       JN=N
01500       IF(VECRD(NM1,I).NE.0..OR.VECRD(N,I).NE.0.)JN=NM2
01510       DO 112 J=1,JN
01520       READ(5,*)VECRD(J,I),VECID(J,I)
01530   112 CONTINUE
01540   140 CONTINUE
01550       DO 150 I=1,N
```

```
01580       I1=I-1
01590       DO 151 J=1,N
01600       VECRD(J,I)=VECRD(J,I1)
01610       VECID(J,I)=-VECID(J,I1)
01620   151 CONTINUE
01630       DO 152 J=1,2
01640       WR(J,I)=WR(J,I1)
01650       WI(J,I)=-WI(J,I1)
01660   152 CONTINUE
01670       GO TO 150
01680   153 IF(VECRD(NM1,I).NE.0..OR.VECRD(N,I).NE.0.)GO TO 154
01690       CALL PROG2(I,ERD,ETD,WR,WI,VECRD,VECID,A,B,N,N2,OB,LB,YB,
01700+BB,II,IBT,VB,P,C)
01710       GO TO 150
01720   154 CALL PROG4(I,ERD,WR,VECRD,A,B,N,NM2,NM1,OB,LB,YB,BB,II,
01730+IBT,VB,P,C,PP,DV)
01740   150 CONTINUE
01750       DO 160 I=1,N
01760       I1=I-1
01770       IF(KRET(1,I).EQ.0)GO TO 160
01780       DO 161 J=1,N
01790       VECRD(J,I)=VECID(J,I1)
01800   161 CONTINUE
01810       DO 162 J=1,2
01820       WR(J,I)=WI(J,I1)
01830   162 CONTINUE
01840   160 CONTINUE
01850       EPS=1.E-10
01860C
01870C      FIND INVERSE(VECRD)
01880C
01890       CALL MXLNEQ(VECRD,N,N,DET,JRANK,EPS,VECID)
01900       CALL MATMUL(WR,VECRD,WI,2,N,N)
01910       DO 165 I=1,N
01920       WRITE(3,164)(WI(J,I),J=1,2)
01930   164 FORMAT(3E16.8)
01940   165 CONTINUE
01950       CALL MATMUL(B,WI,VECID,N,2,N)
01960       CALL MATADD(A,VECID,VECRD,N,N)
01970       CALL PROG3(N,VECRD,ERD,ETD,Z,IVI,FVI,KRET)
01980       CALL REPLACE(5HTAPE3,5HGAINS,0,0)
01990       RETURN
02000       END
02010C
02020C
02030C.....T(M,P)=A(M,N) * U(N,P)
02040       SUBROUTINE MATMUL(A,U,T,M,N,P)
02050       INTEGER P
02060       DIMENSION A(M,N),U(N,P),T(M,P)
```

```
02090       T(I,J)=0.
02100     1 CONTINUE
02110       DO 2 I=1,M
02120       DO 2 J=1,P
02130       DO 2 K=1,N
02140       T(I,J)=A(I,K)*U(K,J)+T(I,J)
02150     2 CONTINUE
02160       RETURN
02170       END
02180C
02190C
02200C.....C(M,N)=A(M,N) + B(M,N)
02210       SUBROUTINE MATADD(A,B,C,M,N)
02220       DIMENSION A(M,N),B(M,N),C(M,N)
02230       DO 3 I=1,M
02240       DO 3 J=1,N
02250       C(I,J)=A(I,J)+B(I,J)
02260     3 CONTINUE
02270       RETURN
02280       END
02290C
02300C
02310C.....B(M,N)=S * A(M,N)
02320       SUBROUTINE SCAMAT(S,A,B,M,N)
02330       DIMENSION A(M,N),B(M,N)
02340       DO 8 I=1,M
02350       DO 8 J=1,N
02360       B(I,J)=S*A(I,J)
02370     8 CONTINUE
02380       RETURN
02390       END
02400C
02410C
02420C.....V(N,M)= A(M,N) TRANSPOSED
02430       SUBROUTINE MATTRA(A,V,M,N)
02440       DIMENSION A(M,N),V(N,M)
02450       DO 14 I=1,M
02460       DO 14 J=1,N
02470       V(J,I)=A(I,J)
02480    14 CONTINUE
02490       RETURN
02500       END
02510C
02520C
02530C
02540       SUBROUTINE PROG3(N,AA,WR,WI,Z,IV1,FV1,KRET)
02550       DIMENSION AA(N,N),WR(N),WI(N),Z(N,N),IV1(N),FV1(N),KRET(2,N)
02560C
02570C      CALCULATE CLOSED LOOP EIGENVALUES AND EIGENVECTORS
```

```
      WRITE(6,23)
   23 FORMAT(41X,'CLOSED LOOP EIGENVALUES'/31X,'REAL PART ',20X,
     +'IMAGINARY PART')
      DO 30 I=1,N
      WRITE(6,25)I,WR(I),WI(I)
   25 FORMAT(20X,I2,'.',2X,E15.4,15X,E15.4)
   30 CONTINUE
      WRITE(6,55)
   55 FORMAT(1X,'DO YOU WANT THE CLOSED LOOP EIGENVECTORS OUTPUT? YES(1)
     +,NO(0)')
      READ(5,*)MM
      IF(MM.EQ.0)GO TO 60
      WRITE(6,64)
   64 FORMAT(1X,'INPUT PRINT(1) OR SUPPRESS(0) FOR EACH EIGENVECTOR')
      READ(5,*)(KRET(1,J),J=1,N)
      WRITE(6,34)
   34 FORMAT(41X,'CLOSED LOOP EIGENVECTORS'/31X,'REAL PART',20X,
     +'IMAGINARY PART')
      IFLAG=0
      DO 50 J=1,N
      IF(KRET(1,J).EQ.0)GO TO 49
      IF(WI(J).NE.0.0)GO TO 35
      DO 37 I=1,N
      IF(I.NE.1)GO TO 36
      WRITE(6,38)J,Z(1,J)
   38 FORMAT(//20X,I2,'.',2X,E15.4,20X,'0.0 ')
      GO TO 37
   36 WRITE(6,39)Z(I,J)
   39 FORMAT(25X,E15.4,20X,'0.0')
   37 CONTINUE
      GO TO 50
   35 IF(IFLAG.EQ.1)GO TO 40
      IFLAG=1
      DO 41 I=1,N
      IF(I.NE.1)GO TO 42
      WRITE(6,43)J,Z(1,J),Z(1,J+1)
   43 FORMAT(//20X,I2,'.',2X,E15.4,15X,E15.4)
      GO TO 41
   42 WRITE(6,44)Z(I,J),Z(I,J+1)
   44 FORMAT(25X,E15.4,15X,E15.4)
   41 CONTINUE
      GO TO 50
   40 J1=J-1
      WRITE(6,45)I,J1
   45 FORMAT(//20X,I2,'.',5X,'CONJUGATE OF EIGENVECTOR ',1X,I2,'.')
   49 IFLAG=0
   50 CONTINUE
   60 RETURN
      END
```

```
03100C
03110      SUBROUTINE PROG2(ID,ERD,ETD,WR,WI,VECRD,VECID,A,B,N,N2,QR,LB,YB,
03120+BB,II,LBT,VB,P,C)
03130      DIMENSION ERD(N),ETD(N),WR(2,N),WI(2,N),VECRD(N,N),VECID(N,N),
03140+A(N,N),B(N,2),QB(N2,N2),WB(4,1),VB(N2,1),YB(N2,N2),BB(N2,4),
03150+C(N,N),R(4,4),P(N2,1),T(4,1),RR(4,4)
03160      REAL II(N,N),LB(N2,4),LBT(4,N2),LAMR,LAMI
03170      LAMR=ERD(ID)
03180      LAMI=ETD(ID)
03190      DO 5 I=1,N
03200      VB(I,1)=VECRD(I,ID)
03210      VB(I+N,1)=VECID(I,ID)
03220    5 CONTINUE
03230      DO 10 I=1,N
03240      DO 10 J=1,N
03250      II(I,J)=0.
03260      IF(I.EQ.J)II(I,J)=1.
03270   10 CONTINUE
03280      CALL SCAMAT(LAMR,II,C,N,N)
03290      DO 20 I=1,N
03300      DO 20 J=1,N
03310      YB(I,J)=C(I,J)-A(I,J)
03320      YB(I+N,J+N)=YB(I,J)
03330   20 CONTINUE
03340      CALL SCAMAT(LAMI,II,C,N,N)
03350      DO 30 I=1,N
03360      DO 30 J=1,N
03370      YB(I,J+N)=-C(I,J)
03380      YB(I+N,J)=C(I,J)
03390   30 CONTINUE
03400      EPS=1.E-10
03410C
03420C     FIND INVERSE(YB)
03430C
03440      CALL MXLNEQ(YB,N2,N2,DET,JRANK,EPS,QB)
03450      DO 40 I=1,N
03460      DO 40 J=1,2
03470      BB(I,J)=B(I,J)
03480      BB(I+N,J)=0.
03490      BB(I,J+2)=0.
03500      BB(I+N,J+2)=B(I,J)
03510   40 CONTINUE
03520      CALL MATMUL(YB,BB,LB,N2,N2,4)
03530      DO 42 I=1,N2
03540      DO 42 J=1,N2
03550      QB(I,J)=0.
03560   42 CONTINUE
03570      WRITE(6,41)ID
03580   41 FORMAT(1X,'INPUT REAL PART OF THE DIAGONAL OF THE QR MATRIX FOR
03590+EIGENVECTOR ',I2,'.')
```

```
03620C
03630       READ(5,*)(QB(J,J),J=1,N)
03640       WRITE(6,43)ID
03650    43 FORMAT(1X,'INPUT IMAGINARY PART OF THE DIAGONAL OF THE QB MATRIX
03660+FOR EIGENVECTOR ',I2,'.')
03670       READ(5,*)(QB(J+N,J+N),J=1,N)
03680       CALL MATMUL(QB,LB,BB,N2,N2,4)
03690       CALL MATTRA(LB,LBT,N2,4)
03700       CALL MATMUL(LBT,BB,R,4,N2,4)
03710C
03720C      FIND INVERSE(R)
03730C
03740       CALL MXLNEQ(R,4,4,DET,JRANK,EPS,RR)
03750       CALL MATMUL(QB,VR,P,N2,N2,1)
03760       CALL MATMUL(LBT,P,T,4,N2,1)
03770       CALL MATMUL(R,T,WR,4,4,1)
03780       CALL MATMUL(LB,WB,VR,N2,4,1)
03790       DO 60 I=1,N
03800       VECRD(I,ID)=VB(I,1)
03810       VECID(I,ID)=VB(I+N,1)
03820    60 CONTINUE
03830       DO 70 I=1,2
03840       WR(I,ID)=WR(I,1)
03850       WI(I,ID)=WR(I+2,1)
03860    70 CONTINUE
03870       RETURN
03880       END
03890C
03900C
03910       SUBROUTINE PROG4(ID,ERD,WR,VECRD,A,B,N,NM2,NM1,QB,LB,YB,
03920+BB,II,LBT,VB,PP,C,P,DV)
03930C
03940C      THIS ROUTINE DETERMINES THE CLOSED LOOP ATTAINABLE
03950C      EIGENVECTORS FOR THE UNCONTROLLABLE GUST MODES
03960C
03970       DIMENSION ERD(N),WR(2,N),VECRD(N,N),A(N,N),B(N,2),QB(NM2,
03980+NM2),WB(2,1),VB(NM2,1),YB(NM2,NM2),BB(NM2,2),C(N,N),R(2,2),
03990+PP(NM2,1),T(2,1),RR(2,2),P(NM2,2),VD(2,1),DV(NM2,1)
04000       REAL II(N,N),LB(NM2,2),LBT(2,NM2),LAMD
04010       LAMD=ERD(ID)
04020       DO 5 I=1,NM2
04030       VB(I,1)=VECRD(I,ID)
04040     5 CONTINUE
04050       VD(1,1)=VECRD(NM1,ID)
04060       VD(2,1)=VECRD(N,ID)
04070       DO 10 I=1,N
04080       DO 10 J=1,N
04090       II(I,J)=0.
04100       IF(I.EQ.J)II(I,I)=1.
```

```
04130       DO 20 I=1,NM2
04140       P(I,1)=C(I,NM1)-A(I,NM1)
04150       P(I,2)=C(I,N)-A(I,N)
04160       DO 20 J=1,NM2
04170       YB(I,J)=C(I,J)-A(I,J)
04180    20 CONTINUE
04190       EPS=1.E-10
04200C
04210C      FIND INVERSE(YB)
04220C
04230       CALL MXLNEQ(YB,NM2,NM2,DET,JRANK,EPS,QB)
04240       DO 40 I=1,NM2
04250       DO 40 J=1,2
04260       BB(I,J)=R(I,J)
04270    40 CONTINUE
04280       CALL MATMUL(YB,P,LB,NM2,NM2,2)
04290       CALL MATMUL(LB,VD,DV,NM2,2,1)
04300       CALL MATMUL(YB,BB,LB,NM2,NM2,2)
04310       CALL MATADD(VB,DV,VB,NM2,1)
04320       DO 42 I=1,NM2
04330       DO 42 J=1,NM2
04340       QB(I,J)=0.
04350    42 CONTINUE
04360       WRITE(6,41)ID
04370    41 FORMAT(1X,'INPUT REAL PART OF THE DIAGONAL OF THE QR MATRIX FOR
04380+EIGENVECTOR ',I2,'.')
04390       READ(5,*)(QB(J,J),J=1,NM2)
04400       CALL MATMUL(QB,LB,BB,NM2,NM2,2)
04410       CALL MATTRA(LB,LBT,NM2,2)
04420       CALL MATMUL(LBT,BB,R,2,NM2,2)
04430C
04440C      FIND INVERSE(R)
04450C
04460       CALL MXLNEQ(R,2,2,DET,IRANK,EPS,RR)
04470       CALL MATMUL(QB,VB,PP,NM2,NM2,1)
04480       CALL MATMUL(LBT,PP,T,2,NM2,1)
04490       CALL MATMUL(R,T,WB,2,2,1)
04500       CALL MATMUL(LB,WB,VB,NM2,2,1)
04510       CALL SCAMAT(-1.,DV,DV,NM2,1)
04520       CALL MATADD(VB,DV,VB,NM2,1)
04530       DO 60 I=1,NM2
04540       VECRD(I,ID)=VB(I,1)
04550    60 CONTINUE
04560       DO 70 I=1,2
04570       WR(I,ID)=WB(I,1)
04580    70 CONTINUE
04590       RETURN
04600       END
```

```
00100      PROGRAM COV(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,FILE3,
00110+TAPE1=FILE3,GAINS,TAPE3=GAINS,TAPE2)
00120C*****************************************************
00130C     THIS PROGRAM INPUTS THE STATE MATRICES
00140C     (OUTPUT OF FLUTTER) A, B, W, FEEDBACK
00150C     GAINS(OUTPUT OF MODAL), INTENSITY OF THE
00160C     GUST NOISE AND OUTPUTS THE CLOSED LOOP
00170C     STATE COVARIANCE MATRIX, RMS CONTROL SURFACE
00180C     RATES AND DEFLECTIONS IN DEGREES.(NOTE: INPUT
00190C     ZERO FEEDBACK GAINS FOR OPEN LOOP RESULTS)
00200C*****************************************************
00210      DIMENSION A(42,42),WI(2,42),Z(42,42),
00220+FV1(42),VECRD(42,42),VECID(42,42),W(42,1),
00230+B(42,2)
00240      WRITE(6,10)
00250   10 FORMAT(1X,'INPUT ORDER OF SYSTEM')
00260      READ(5,*)N
00270      CALL PROG1(N,A,WI,Z,FV1,VECRD,VECID,
00280+W,B)
00290      STOP
00300      END
00310C
00320C
00330      SUBROUTINE PROG1(N,A,WI,Z,FV1,VECRD,VECID,
00340+W,B)
00350      DIMENSION A(N,N),WI(2,N),Z(N,N),FV1(N),
00360+VECRD(N,N),VECID(N,N),W(N,1),B(N,2)
00370      DIMENSION PP(42,42),DD(42,42),WI(1,42)
00380      REAL II(42,42)
00390      CALL GETPF(5HTAPE1,5HFILE3,0,0)
00400C
00410C     READ IN STATE MATRICES OUTPUT FROM PROGRAM: FLUTTER
00420C
00430      DO 13 I=1,N
00440      READ(1,15)W(I,1)
00450   15 FORMAT(3E16.8)
00460   13 CONTINUE
00470      DO 16 I=1,N
00480      DO 16 J=1,N
00490      READ(1,15)A(I,J)
00500   16 CONTINUE
00510      DO 17 I=1,N
00520      READ(1,15)B(I,1),B(I,2)
00530   17 CONTINUE
```

```
00560C      READ IN FEEDBACK GAINS OUTPUT FROM PROGRAM: MODAL
00570C
00580       DO 24 I=1,N
00590       READ(3,*)(WI(J,I),J=1,2)
00600    24 CONTINUE
00610       CALL MATMUL(B,WI,VECID,N,2,N)
00620       CALL MATADD(A,VECID,VECRD,N,N)
00630       CALL PROG4(N,VECRD,VECID,A,Z,II,PP,DD,EV1,W,WT)
00640       RETURN
00650       END
00660C
00670C
00680C.....T(M,P)=A(M,N) * U(N,P)
00690       SUBROUTINE MATMUL(A,U,T,M,N,P)
00700       INTEGER P
00710       DIMENSION A(M,N),U(N,P),T(M,P)
00720       DO 1 I=1,M
00730       DO 1 J=1,P
00740       T(I,J)=0.
00750     1 CONTINUE
00760       DO 2 I=1,M
00770       DO 2 J=1,P
00780       DO 2 K=1,N
00790       T(I,J)=A(I,K)*U(K,J)+T(I,J)
00800     2 CONTINUE
00810       RETURN
00820       END
00830C
00840C
00850C.....C(M,N)=A(M,N) + B(M,N)
00860       SUBROUTINE MATADD(A,B,C,M,N)
00870       DIMENSION A(M,N),B(M,N),C(M,N)
00880       DO 3 I=1,M
00890       DO 3 J=1,N
00900       C(I,J)=A(I,J)+B(I,J)
00910     3 CONTINUE
00920       RETURN
00930       END
00940C
00950C
00960C.....B(M,N)=S * A(M,N)
00970       SUBROUTINE SCAMAT(S,A,B,M,N)
00980       DIMENSION A(M,N),B(M,N)
00990       DO 8 I=1,M
01000       DO 8 J=1,N
01010       B(I,J)=S*A(I,J)
01020     8 CONTINUE
01030       RETURN
01040       END
```

```
01070C.....V(N,M)= A(M,N) TRANSPOSED
01080      SUBROUTINE MATTRA(A,V,M,N)
01090      DIMENSION A(M,N),V(N,M)
01100      DO 14 I=1,M
01110      DO 14 J=1,N
01120      V(J,I)=A(I,J)
01130   14 CONTINUE
01140      RETURN
01150      END
01160C
01170C
01180C
01190      SUBROUTINE PROG4(N,A,X,Q,XN,E,P,DX,WA,D,DT)
01200      DIMENSION A(N,N),X(N,N),Q(N,N),XN(N,N),E(N,N),P(N,N),DX(N,N),
01210+WA(N),D(N,1),DT(1,N)
01220      WRITE(6,10)
01230   10 FORMAT(1X,'INPUT THE INTENSITY OF THE GUST NOISE, R.')
01240      READ(5,*)R
01250      CALL MATTRA(D,DT,N,1)
01260      CALL SCAMAT(R,D,D,N,1)
01270      CALL MATMUL(D,DT,Q,N,1,N)
01280      CALL CAL(A,Q,XN,N,15,2,X,E,P,DX,WA)
01290      WRITE(6,35)
01300      NN=N-9
01310      ZZ=57.3*SQRT(XN(NN+1,NN+1))
01320      WRITE(6,38)ZZ
01330      ZZ=57.3*SQRT(XN(NN+2,NN+2))
01340      WRITE(6,39)ZZ
01350      ZZ=57.3*SQRT(XN(NN+4,NN+4))
01360      WRITE(6,40)ZZ
01370       ZZ=57.3*SQRT(XN(NN+5,NN+5))
01380       WRITE(6,41)ZZ
01390   38 FORMAT(1X,'OUTBOARD DEFLECTION =  ,E15.4)
01400   39 FORMAT(1X,'OUTBOARD RATE = ',E15.4)
01410   40 FORMAT(1X,'INBOARD DEFLECTION = ',E15.4)
01420   41 FORMAT(1X,'INBOARD RATE = ',E15.4)
01430   35 FORMAT(1X,'RMS OF CONTROL RESPONSE'//)
01440      DO 24 I=1,N
01450      DO 24 J=1,N
01460      WRITE(2,164)XN(I,J)
01470  164 FORMAT(E16.8)
01480   24 CONTINUE
01490      CALL REPLACE(5HTAPE2,5HCOVST,0,0)
01500      RETURN
01510      END
01520C
01530C
01540      SUBROUTINE CAL(A,Q,XN,N,IMAX,IT,X,E,P,DX,WA)
01550C
```

```
01580      DIMENSION A(N,N),D(N,N),XN(N,N)
01590      DIMENSION X(N,N),E(N,N),P(N,N),DX(N,N),WA(N)
01600      TR=0.
01610      DO 300 I=1,N
01620  300 TR=TR+A(I,I)
01630      FN=N
01640      IF(TR)301,302,301
01650  301 ALF=ABS(TR)/FN
01660      GOTO 303
01670  302 ALF=1.
01680  303 CONTINUE
01690      EE=.01
01700      NC=N*(N+1)
01710      NC=NC/2
01720      DO 60 I=1,N
01730      DO 63 J=1,N
01740      GOTO(61,62),TT
01750   61 P(I,J)=A(I,J)
01760      X(I,J)=A(I,J)
01770      GOTO 63
01780   62 P(I,J)=A(J,I)
01790      X(I,J)=A(J,I)
01800   63 CONTINUE
01810      P(I,I)=P(I,I)-ALF
01820      X(I,I)=X(I,I)-ALF
01830   60 CONTINUE
01840C
01850C     FIND INVERSE(P)
01860C
01870      CALL MXLNEQ(P,N,N,DET,N,1.E-14,WA)
01880      DO 4 I=1,N
01890      DO 4 J=1,N
01900      E(I,J)=0.
01910      DO 4 K=1,N
01920    4 E(I,J)=E(I,J)+P(K,I)*D(K,J)*2.*ALF
01930      DO 5 I=1,N
01940      DO 5 J=1,N
01950      XN(I,J)=0.
01960      DO 5 K=1,N
01970    5 XN(I,J)=XN(I,J)+E(I,K)*P(K,J)
01980      DO 7 I=1,N
01990      DO 8 J=1,N
02000    8 P(I,J)=P(I,J)+2.*ALF
02010    7 P(I,I)=P(I,I)+1.
02020      ITER=0
02030  100 CONTINUE
02040      DO 9 I=1,N
02050      DO 9 J=1,N
02060      E(I,J)=0.
```

```
02080     9 E(I,J)=E(I,J)+P(K,J)*XN(K,J)
02090       DO 10 I=1,N
02100       DO 10 J=I,N
02110       DX(I,J)=0.
02120       DO 10 K=1,N
02130    10 DX(I,J)=DX(I,J)+E(I,K)*P(K,J)
02140       DO 20 I=1,N
02150       DO 20 J=1,N
02160       E(I,J)=P(I,J)
02170    20 X(I,J)=P(I,J)
02180       DO 12 I=1,N
02190       DO 12 J=I,N
02200       XN(I,J)=XN(I,J)+DX(I,J)
02210    12 XN(J,I)=XN(I,J)
02220       ICOT=0
02230       DO 15 I=1,N
02240       DO 15 J=I,N
02250       IF(XN(I,J))201,14,201
02260   201 RAT=ABS(DX(I,J)/XN(I,J))
02270       IF(RAT-EE)14,14,70
02280    14 ICOT=ICOT+1
02290    15 CONTINUE
02300    70 CONTINUE
02310       IF(ICOT-NC)16,18,16
02320    16 DO 17 I=1,N
02330       DO 17 J=1,N
02340       P(I,J)=0.
02350       DO 17 K=1,N
02360    17 P(I,J)=P(I,J)+E(I,K)*X(K,J)
02370    18 ITER=ITER+1
02380       IF(ICOT-NC)40,50,40
02390    40 IF(ITER-IMAX)100,50,50
02400    50 CONTINUE
02410       PRINT 600,ITER
02420   600 FORMAT(10X,'INNER LOOP TERMINATED AT ITER = ',I2)
02430       RETURN
02440       END
```

```
 82/12/15
MNFTS     PROGRAM   LOADS

00100      PROGRAM LOADS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,COVST,
00110+TAPE3=COVST,LMAT,TAPE2=LMAT)
00120C*****************************************************************
00130C     THIS PROGRAM INPUTS CLOSED LOOP STATE COVARIANCE MATRIX
00140C     (OUTPUT FROM COV), LOADS INFLUENCE MATRIX AND OUTPUTS
00150C     THE RMS SHEAR, RMS BENDING MOMENT, AND RMS TORQUE AT
00160C     VARIOUS STATIONS ALONG THE WING SPAN.
00170C*****************************************************************
00180      REAL XN(42,42),T(10,42),LT(9,42),LTT(42,9),TEMP(42,9)
00190      DIMENSION KRET(8)
00200      WRITE(6,10)
00210   10 FORMAT(1X,'INPUT WHICH STATES WHERE RETAINED(1) OR DELETED(0)?')
00220      READ(5,*)(KRET(J),J=1,8)
00230      KSUM=0
00240      DO 20 I=1,8
00250      KSUM=KSUM+KRET(I)
00260   20 CONTINUE
00270      WRITE(6,11)
00280   11 FORMAT(1X,'INPUT THE NUMBER OF LAG STATES?')
00290      READ(5,*)M
00300      KST=KSUM*(2+M)
00310      KKSUM=KST+9
00320      CALL PROG1(KST,KKSUM,KRET,XN,T,LT,LTT,TEMP)
00330      STOP
00340      END
00350C
00360C
00370      SUBROUTINE PROG1(KST,KKSUM,KRET,XN,T,LT,LTT,TEMP)
00380      REAL LBM(9,10),LSH(9,10),LTO(9,10),XN(KKSUM,KKSUM),T(10,KKSUM),
00390+LT(9,KKSUM),LTT(KKSUM,9),TEMP(KKSUM,9),COV(9,9),BM(9),SH(9),TO(9)
00400      DIMENSION KRET(8)
00410      CALL GETPF(5HTAPE2,4HLMAT,0,0)
00420      DO 10 I=1,10
00430      DO 10 J=1,9
00440C
00450C     READ IN LOADS INFLUENCE MATRIX
00460C
00470      READ(2,15)LBM(J,I),LSH(J,I),LTO(J,I)
00480   15 FORMAT(3E16.8)
00490   10 CONTINUE
00500      CALL GETPF(5HTAPE3,5HCOVST,0,0)
00510      DO 20 I=1,KKSUM
00520      DO 20 J=1,KKSUM
```

```
00550C
00560       READ(3,15)XN(I,J)
00570    20 CONTINUE
00580       DO 40 I=1,10
00590       DO 40 J=1,KKSUM
00600       T(I,J)=0.
00610    40 CONTINUE
00620       J=0
00630       DO 50 I=1,8
00640       IF(KRET(I).EQ.0)GO TO 50
00650       J=J+1
00660       T(I,J)=1.
00670    50 CONTINUE
00680       T(9,KST+1)=.514
00690       T(10,KST+4)=.518
00700       CALL MATMUL(LBM,T,LT,9,10,KKSUM)
00710       CALL MATTRA(LT,LTT,9,KKSUM)
00720       CALL MATMUL(XN,LTT,TEMP,KKSUM,KKSUM,9)
00730       CALL MATMUL(LT,TEMP,COV,9,KKSUM,9)
00740       DO 60 I=1,9
00750       BM(I)=SQRT(COV(I,I))
00760    60 CONTINUE
00770       CALL MATMUL(LSH,T,LT,9,10,KKSUM)
00780       CALL MATTRA(LT,LTT,9,KKSUM)
00790       CALL MATMUL(XN,LTT,TEMP,KKSUM,KKSUM,9)
00800       CALL MATMUL(LT,TEMP,COV,9,KKSUM,9)
00810       DO 70 I=1,9
00820       SH(I)=SQRT(COV(I,I))
00830    70 CONTINUE
00840       CALL MATMUL(LTQ,T,LT,9,10,KKSUM)
00850       CALL MATTRA(LT,LTT,9,KKSUM)
00860       CALL MATMUL(XN,LTT,TEMP,KKSUM,KKSUM,9)
00870       CALL MATMUL(LT,TEMP,COV,9,KKSUM,9)
00880       DO 80 I=1,9
00890       TQ(I)=SQRT(COV(I,I))
00900    80 CONTINUE
00910       WRITE(6,90)
00920    90 FORMAT(35X,'RMS LOADS'/1X,'BEND MOM',27X,'SHEAR',27X,'TORQUE'//)
00930       DO 100 I=1,9
00940   110 FORMAT(5X,E16.4,10X,E16.4,10X,E16.4)
00950       WRITE(6,110)BM(I),SH(I),TQ(I)
00960   100 CONTINUE
00970       RETURN
00980       END
00990C
01000C
01010       SUBROUTINE MATMUL(A,B,T,M,N,P)
01020       INTEGER P
01040       DIMENSION A(M,N),B(N,P),T(M,P)
```

```
01060       T(I,J)=0.
01070     1 CONTINUE
01080       DO 2 I=1,M
01090       DO 2 J=1,P
01100       DO 2 K=1,N
01110       T(I,J)=A(I,K)*U(K,J)+T(I,J)
01120     2 CONTINUE
01130       RETURN
01140       END
01150C
01160C
01170       SUBROUTINE MATTRA(A,V,M,N)
01180       DIMENSION A(M,N),V(N,M)
01190       DO 14 I=1,M
01200       DO 14 J=1,N
01210       V(J,I)=A(I,J)
01220    14 CONTINUE
01230       RETURN
01240       END
```

```
00100      PROGRAM FEEDBKE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,ONELEE,
00110+TAPE2=ONELEE,MASS,TAPE1=MASS,GAINL,TAPE3=GAINL,TAPE4)
00120C*****************************************************************
00130C     THIS PROGRAM INPUTS THE STRUCTURAL MASS,STRUCTURAL STIFFNESS,
00140C     STRUCTURAL DAMPING, AERODYNAMIC INFLUENCE MATRICES, FLIGHT
00150C     CONDITION, FEEDBACK GAINS(OUTPUT OF MODAL) AND OUTPUTS THE
00160C     CLOSED LOOP EIGENVALUES.  THIS IS USEFUL TO STUDY OFF DESIGN
00170C     PERFORMANCE.
00180C*****************************************************************
00190      REAL MXX(8,8),CS(8,8),KS(8,8),CT(8,8),KT(8,8),CC(8,8),
00200+KK(8,8),A2X(8,8),A1X(8,8),A0X(8,8),MINV(8,8),A2X0(8,8)
00210+,A1X0(8,8),A0X0(8,8),A1(16,16),A2(32,32),00(8,2),DUM1(8,7),
00220+DUM2(8,7),DUM3(8,2),DUM4(8,7),DUM5(8,2),DUM6(8,2),DUM7(8,1)
00230      REAL DUM8(8,1),A0U(8,2),A1U(8,2),EI(8,2,4),R(8,2),
00240+ETJG(8,7,4),FI(8,4),A3(40,40),B3(40,2),B2(32,2),II(8,8),
00250+DI(8,8,4),KT(4),A1D(8,1),A0D(8,1),T(8,1),U(8,1),A4(42,42),
00260+B4(42,2),W(42,1),B1(16,2),A5(24,24),B5(24,2)
00270      WRITE(6,8)
00280    8 FORMAT(1X,'INPUT THE ORDER OF THE STRUCTURAL-RIGID BODY STATE VECTO
00290+R(N)'/)
00300      READ(5,*)N
00310      WRITE(6,10)
00320   10 FORMAT(1X,'INPUT VELOCITY(V), CHORD(C), DYNAMIC PRESSURE(Q), NUMBE
00330+R OF LAG STATES(L)'/)
00340C
00350C     INPUT FLIGHT CONDITIONS
00360C
00370      READ(5,*)V,C,Q,L
00380      DO 20 I=1,L
00390      WRITE(6,25)I
00400C
00410C     INPUT AERODYNAMIC LAG FREQUENCIES
00420C
00430      READ(5,*)KT(I)
00440   20 CONTINUE
00450   25 FORMAT(1X,'INPUT KT(',I1,')'/)
00460      N2=2*N
00470      KEND=N*(L+2)
00480      KENDD=KEND+7
00490      KEND2=KENDD+2
00500      K5=2*N+7
00510      CALL PROG1(N,N2,KEND,KENDD,KEND2,K5,V,C,Q,L,MXX,CS,KS,CT,KT,
00520+CC,KK,A2X,A1X,A0X,MINV,A2X0,A1X0,A0X0,A1,A2,00,DUM1,DUM2,DUM3,DUM4
00530+,DUM5,DUM6,DUM7,DUM8,A0U,A1U,EI,R,ETJG,II,A3,B3,B2,FI,DI,KT,A1D,
```

```
00550       STOP
00560       END
00570C
00580C
00590       SUBROUTINE PROG1(N,N2,KEND,KEND0,KEND2,K5,U,C,Q,L,MXX,CS,KS,CT,KT,
00600+CC,KK,A2X,A1X,A0X,MINV,A2XQ,A1XQ,A0XQ,A1,A2,QQ,DUM1,DUM2,DUM3,DUM4
00610+,DUM5,DUM6,DUM7,DUM8,A0U,A1U,ET,R,ETJG,II,A3,B3,B2,FI,DI,KI,A1D,
00620+A0D,T,U,A4,R1,B4,W,A5,B5)
00630       REAL LAMDA,LL
00640       INTEGER FLAG
00650       COMMON KRET(42)
00660       REAL MXX(N,N),CS(N,N),KS(N,N),CT(N,N),KT(N,N),CC(N,N),KK(N,N),
00670+A2X(N,N),A1X(N,N),A0X(N,N),MINV(N,N),A2XQ(N,N),A1XQ(N,N),A0XQ(N,N)
00680+,A1(N2,N2),A2(KEND,KEND),QQ(N,2),DUM1(N,7),DUM2(N,7),DUM3(N,2),
00690+DUM4(N,7),DUM5(N,2),DUM6(N,2),DUM7(N,1),DUM8(N,1),A0U(N,2)
00700       REAL A1U(N,2),
00710+EI(N,2,L),G(7,7),H(7,2),R(N,2),JG(2,7),ETJG(N,7,L),FI(N,L),
00720+JH(2,2),A3(KEND0,KEND0),B3(KEND0,2),B2(KEND,2),II(N,N),DI(N,N,L)
00730       REAL KI(L),A1D(N,1),A0D(N,1),T(N,1),U(N,1),A4(KEND2,KEND2),
00740+B1(N2,2),B4(KEND2,2),W(KEND2,1),
00750+A5(K5,K5),B5(K5,2),JJ(2,7)
00760       REAL AA(42,42),BB(42,2)
00770       CALL GETPF(5HTAPE1,4HMASS,0,0)
00780       DO 1 I=1,N
00790       DO 1 J=1,N
00800C
00810C      READ IN STRUCTURAL DATA
00820C
00830       READ(1,101)MXX(I,J),CS(I,J),KS(I,J)
00840     1 CONTINUE
00850       DO 3 I=1,7
00860       DO 3 J=1,7
00870C
00880C      READ IN ACTUATOR DATA
00890C
00900       READ(1,101)G(I,J)
00910     3 CONTINUE
00920       DO 4 I=1,7
00930       READ(1,101)(H(I,J),J=1,2)
00940     4 CONTINUE
00950       DO 5 J=1,7
00960       READ(1,101)(JJ(I,J),I=1,2)
00970     5 CONTINUE
00980       CALL GETPF(5HTAPE2,6HONELEE,0,0)
00990       DO 430 K=1,L
01000       DO 410 J=1,N
01010       DO 410 I=1,N
01020C
01030C      READ IN AERODYNAMIC INFLUENCE DATA
01040C
```

```
01070      DO 420 I=1,2
01080      DO 420 J=1,N
01090      READ(2,101)A0U(J,I),A1U(J,I),ET(J,I,K)
01100  420 CONTINUE
01110      DO 430 J=1,N
01120      READ(2,101)A0D(J,K),A1D(J,K),FI(J,K)
01130      FI(J,K)=FI(J,K)/V
01140  430 CONTINUE
01150  101 FORMAT(3E16.8)
01160  109 FORMAT(4E16.8)
01170      EPS=1.E-10
01180      TEMP=Q*(C/2./V)**2
01190      TEMP2=Q*C/2./V
01200      CALL SCAMAT(TEMP,A2X,A2XQ,N,N)
01210      CALL MATADD(MXX,A2XQ,MINV,N,N)
01220C
01230C     FIND INVERSE(MINV)
01240C
01250      CALL MXLNEQ(MINV,N,N,DET,JRANK,EPS,A2XQ,0)
01260      CALL SCAMAT(TEMP2,A1X,A1XQ,N,N)
01270      CALL MATADD(CS,A1XQ,CT,N,N)
01280      CALL SCAMAT(Q,A0X,A0XQ,N,N)
01290      CALL MATADD(KS,A0XQ,KT,N,N)
01300      CALL SCAMAT(-1.,MINV,MINV,N,N)
01310      CALL MATMUL(MINV,KT,KK,N,N,N)
01320      CALL MATMUL(MINV,CT,CC,N,N,N)
01330      DO 15 I=1,N
01340      DO 14 J=1,N
01350      II(I,J)=0.
01360   14 CONTINUE
01370      II(I,I)=1.0
01380   15 CONTINUE
01390      N21=N2+1
01400      DO 18 I=1,N
01410      DO 18 J=1,N
01420      A1(I,J)=0.
01430      A1(I,J+N)=II(I,J)
01440      A1(I+N,J)=KK(I,J)
01450      A1(I+N,J+N)=CC(I,J)
01460   18 CONTINUE
01470      IF(I.EQ.0)GO TO 102
01480      DO 20 I=1,KEND
01490      DO 20 J=1,KEND
01500      A2(I,J)=0.
01510   20 CONTINUE
01520      DO 25 I=1,N
01530      DO 25 J=1,N
01540      A2(I,J)=A1(I,J)
01550      A2(I+N,J)=A1(I+N,J)
```

```
      DO 25 K=1,L
      KN=K*N
      A2(I+N,KN+N+J)=MINV(I,J)
      A2(KN+N+I,J+N)=DI(I,J,K)
      A2(KN+N+I,KN+N+I)=-2.*U*KI(K)/C
   25 CONTINUE
      SCA=0*C/2./U
      CALL SCAMAT(SCA,A1U,QQ,N,2)
      CALL SCAMAT(0,A0U,R,N,2)
      DO 36 I=1,KENDD
      DO 36 J=1,KENDD
      A3(I,J)=0.
   36 CONTINUE
      DO 37 I=1,KEND
      DO 37 J=1,KEND
      A3(I,J)=A2(I,J)
   37 CONTINUE
      DO 38 I=1,7
      DO 38 J=1,7
      A3(KEND+I,KEND+J)=G(I,J)
   38 CONTINUE
      CALL MATMUL(JJ,G,JG,2,7,7)
      CALL MATMUL(QQ,JG,DUM1,N,2,7)
      CALL MATMUL(R,JJ,DUM2,N,2,7)
      CALL MATADD(DUM2,DUM1,DUM2,N,7)
      CALL MATMUL(MINV,DUM2,DUM1,N,N,7)
      DO 40 K=1,L
      DO 39 I=1,N
      DO 39 J=1,2
      DUM3(I,J)=EI(I,J,K)
   39 CONTINUE
      CALL MATMUL(DUM3,JG,DUM4,N,2,7)
      DO 40 I=1,N
      DO 40 J=1,7
      ETJG(I,J,K)=DUM4(I,J)
   40 CONTINUE
      DO 45 I=1,N
      DO 45 J=1,7
      A3(I+N,J+KEND)=DUM1(I,J)
      DO 45 K=1,L
      KN=K*N
      A3(KN+N+I,J+KEND)=ETJG(I,J,K)
   45 CONTINUE
      CALL MATMUL(JJ,H,JH,2,7,2)
      CALL MATMUL(QQ,JH,DUM3,N,2,2)
      CALL MATMUL(MINV,DUM3,DUM5,N,N,2)
      DO 47 K=1,L
      DO 46 I=1,N
      DO 46 J=1,2
```

```
      CALL MATMUL(DUM3,JH,DUM6,N,2,2)
      DO 47 I=1,N
      DO 47 J=1,2
      ETJG(I,J,K)=DUM6(I,J)
   47 CONTINUE
      DO 48 I=1,N
      DO 48 J=1,2
      B3(I,J)=0.
      B3(I+N,J)=DUM5(I,J)
      DO 48 K=1,L
      KN=K*N
      B3(I+KN+N,J)=ETJG(I,J,K)
   48 CONTINUE
      DO 49 I=1,7
      DO 49 J=1,2
      B3(I+KEND,J)=H(I,J)
   49 CONTINUE
      WRITE(6,56)
   56 FORMAT(1X,'INPUT CHARACTERISTIC LENGTH(LL) OF NOISE'/)
      READ(5,*)LL
      TEMP3=TEMP2/V
      CALL SCAMAT(TEMP3,A1D,T,N,1)
      Q1=Q/V
      CALL SCAMAT(Q1,A0D,U,N,1)
      CALL MATMUL(MINV,U,DUM7,N,N,1)
      CALL MATMUL(MINV,T,DUM8,N,N,1)
      DO 58 I=1,KENDD
      DO 58 J=1,2
      B4(I,J)=B3(I,J)
      B4(KENDD+1,J)=0.
      B4(KENDD+2,J)=0.
   58 CONTINUE
      DO 59 I=1,KEND2
      DO 59 J=1,KEND2
      A4(I,J)=0.
   59 CONTINUE
      DO 60 I=1,KENDD
      DO 60 J=1,KENDD
      A4(I,J)=A3(I,J)
   60 CONTINUE
      A4(KENDD+1,KENDD+2)=1.
      A4(KENDD+2,KENDD+1)=-1.001*(V/LL)**2
      A4(KENDD+2,KENDD+2)=-2.001*V/LL
      DO 62 I=1,N
      A4(I+N,KENDD+1)=DUM7(I,1)
      A4(I+N,KENDD+2)=DUM8(I,1)
      DO 62 K=1,L
      KN=K*N
      A4(I+KN+N,KENDD+2)=FT(I,K)
```

```
      DO 64 I=1,KENDD
      W(I,1)=0.
   64 CONTINUE
      XX=1.732*V/LL
      W(KENDD+1,1)=XX
      W(KENDD+2,1)=-2.464*(V/LL)**2
      CALL SCAMAT(XX,DUM8,DUM8,N,1)
      DO 66 I=1,N
      W(I+N,1)=DUM8(I,1)
      DO 66 K=1,L
      KN=K*N
      W(I+N+KN,1)=XX*FI(I,K)
   66 CONTINUE
      FLAG=4
      GO TO 200
  102 SCA=0*C/2./V
      CALL SCAMAT(SCA,AIU,QQ,N,2)
      CALL SCAMAT(Q,AOU,R,N,2)
      DO 106 I=1,K5
      DO 106 J=1,K5
      A5(I,J)=0.
  106 CONTINUE
      CALL MATMUL(JJ,H,JH,2,7,2)
      CALL MATMUL(QQ,JH,DUM3,N,2,2)
      CALL MATMUL(MINV,DUM3,DUM5,N,N,2)
      DO 108 I=1,N
      DO 108 J=1,2
      B5(I,J)=0.
      B5(I+N,J)=DUM5(I,J)
  108 CONTINUE
      DO 110 I=1,7
      DO 110 J=1,2
      B5(I+N2,J)=H(I,J)
  110 CONTINUE
      DO 112 I=1,N2
      DO 112 J=1,N2
      A5(I,J)=A1(I,J)
  112 CONTINUE
      CALL MATMUL(JJ,G,JG,2,7,7)
      CALL MATMUL(QQ,JG,DUM1,N,2,7)
      CALL MATMUL(R,JJ,DUM2,N,2,7)
      CALL MATADD(DUM2,DUM1,DUM2,N,7)
      CALL MATMUL(MINV,DUM2,DUM1,N,N,7)
      DO 114 I=1,N
      DO 114 J=1,7
      A5(I+N,J+N2)=DUM1(I,J)
  114 CONTINUE
      DO 116 I=1,7
      DO 116 J=1,7
      A5(I+N2,J+N2)=G(I,J)
```

```
03100       FLAG=1
03110   200 WRITE(6,201)
03120   201 FORMAT(1X,'INPUT RETAIN(1) OR DELETE(0) FOR EACH STATE MEMBER'/)
03130       KSTATE=N
03140       KSUM=0
03150       READ(5,*)(KRET(J),J=1,KSTATE)
03160       DO 205 I=1,KSTATE
03170       KRET(I+N)=KRET(I)
03180       IF(KRET(I).EQ.1)KSUM=KSUM+1
03190   205 CONTINUE
03200       IF(L.EQ.0)GO TO 207
03210       DO 206 K=1,L
03220       DO 206 I=1,N
03230       KN=N*(K+1)+I
03240       KRET(KN)=KRET(I)
03250   206 CONTINUE
03260       DO 209 I=1,9
03270       KRET(KEND+I)=1
03280   209 CONTINUE
03290   207 GO TO(310,320,330,340,350),FLAG
03300   310 KSUM=2*KSUM
03310       CALL PROG2(FLAG,KSUM,A1,B1,N2,AA,BB)
03320       GO TO 900
03330   320 KSUM=(L+2)*KSUM
03340       CALL PROG2(FLAG,KSUM,A2,B2,KEND,AA,BB)
03350       GO TO 900
03360   330 KSUM=7+(L+2)*KSUM
03370       CALL PROG2(FLAG,KSUM,A3,B3,KEND3,AA,BB)
03380       GO TO 900
03390   340 KSUM=9+(L+2)*KSUM
03400       CALL PROG2(FLAG,KSUM,A4,B4,KEND4,AA,BB)
03410       GO TO 900
03420   350 KSUM=7+2*KSUM
03430       DO 351 I=N21,K5
03440       KRET(I)=1
03450   351 CONTINUE
03460       CALL PROG2(FLAG,KSUM,A5,B5,K5,AA,BB)
03470   900 RETURN
03480       END
03490C
03500C
03510       SUBROUTINE MATDEL(A,N,AA,KSUM)
03520C
03530C      THIS ROUTINE DELETES DESIRED STATES FROM THE A MATRIX
03540C
03550       DIMENSION A(N,N),AA(KSUM,KSUM),JRET(40)
03560       COMMON KRET(42)
03570       J=0
03580       DO 100 I=1,N
03590       IF(KRET(I).EQ.0)GO TO 100
```

```
03620  100 CONTINUE
03630      DO 200 I=1,KSUM
03640      DO 200 J=1,KSUM
03650      AA(I,J)=A(JRET(I),JRET(J))
03660  200 CONTINUE
03670      RETURN
03680      END
03690C
03700C
03710      SUBROUTINE MATDELB(B,N,BB,KSUM)
03720C
03730C     THIS ROUTINE DELETES DESIRED STATES FROM THE B MATRIX
03740C
03750      DIMENSION B(N,2),BB(KSUM,2),JRET(40)
03760      COMMON KRET(42)
03770      J=0
03780      DO 100 I=1,N
03790      IF(KRET(I).EQ.0)GO TO 100
03800      J=J+1
03810      JRET(J)=I
03820  100 CONTINUE
03830      DO 200 I=1,KSUM
03840      DO 200 J=1,2
03850      BB(I,J)=B(JRET(I),J)
03860  200 CONTINUE
03870      RETURN
03880      END
03890C
03900C
03910C.....T(M,P)=A(M,N) * U(N,P)
03920      SUBROUTINE MATMUL(A,U,T,M,N,P)
03930      INTEGER P
03940      DIMENSION A(M,N),U(N,P),T(M,P)
03950      DO 1 I=1,M
03960      DO 1 J=1,P
03970      T(I,J)=0.
03980    1 CONTINUE
03990      DO 2 I=1,M
04000      DO 2 J=1,P
04010      DO 2 K=1,N
04020      T(I,J)=A(I,K)*U(K,J)+T(I,J)
04030    2 CONTINUE
04040      RETURN
04050      END
04060C
04070C
04080C.....C(M,N)=A(M,N) + B(M,N)
04090      SUBROUTINE MATADD(A,B,C,M,N)
04100      DIMENSION A(M,N),B(M,N),C(M,N)
```

```
04130       C(I,J)=A(I,J)+B(I,J)
04140     3 CONTINUE
04150       RETURN
04160       END
04170C
04180C
04190C.....B(M,N)=S * A(M,N)
04200       SUBROUTINE SCAMAT(S,A,B,M,N)
04210       DIMENSION A(M,N),B(M,N)
04220       DO 8 I=1,M
04230       DO 8 J=1,N
04240       B(I,J)=S*A(I,J)
04250     8 CONTINUE
04260       RETURN
04270       END
04280C
04290C
04300C.....V(N,M)= A(M,N) TRANSPOSED
04310       SUBROUTINE MATTRA(A,V,M,N)
04320       DIMENSION A(M,N),V(N,M)
04330       DO 14 I=1,M
04340       DO 14 J=1,N
04350       V(J,I)=A(I,J)
04360    14 CONTINUE
04370       RETURN
04380       END
04390C
04400C
04410       SUBROUTINE PROG2(FLAG,KSUM,A,B,IP,AA,BB)
04420       INTEGER FLAG,IV1(42)
04430       REAL AA(KSUM,KSUM),BB(KSUM,2),A(IP,IP),B(IP,2),FV1(42)
04440       REAL WR(42),WI(42),Z(42,42),G(2,42)
04450       GO TO(310,320,330,340,350),FLAG
04460   100 FORMAT(3E16.8)
04470   310 CALL MATDEL(A,IP,AA,KSUM)
04480       DO 311 I=1,KSUM
04490       DO 311 J=1,KSUM
04500       WRITE(4,100)AA(I,J)
04510   311 CONTINUE
04520       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1,BB,G)
04530       GO TO 900
04540   320 CALL MATDEL(A,IP,AA,KSUM)
04550       DO 321 I=1,KSUM
04560       DO 321 J=1,KSUM
04570       WRITE(4,100)AA(I,J)
04580   321 CONTINUE
04590       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1,BB,G)
04600       GO TO 900
04610   330 CALL MATDEL(A,IP,AA,KSUM)
```

```
04630       WRITE(4,100)I
04640       DO 331 J=1,KSUM
04650       WRITE(4,100)AA(I,J)
04660   331 CONTINUE
04670       DO 332 I=1,KSUM
04680       WRITE(4,100)BB(I,1),BB(I,2)
04690   332 CONTINUE
04700       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1,BB,G)
04710       GO TO 900
04720   340 CALL MATDEL(A,IP,AA,KSUM)
04730       CALL MATDELB(B,IP,BB,KSUM)
04740       DO 341 I=1,KSUM
04750       DO 341 J=1,KSUM
04760       WRITE(4,100)AA(I,J)
04770   341 CONTINUE
04780       DO 342 I=1,KSUM
04790       WRITE(4,100)BB(I,1),BB(I,2)
04800   342 CONTINUE
04810       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1,BB,G)
04820       GO TO 900
04830   350 CALL MATDEL(A,IP,AA,KSUM)
04840       CALL MATDELB(B,IP,BB,KSUM)
04850       DO 351 I=1,KSUM
04860       DO 351 J=1,KSUM
04870       WRITE(4,100)AA(I,J)
04880   351 CONTINUE
04890       DO 352 I=1,KSUM
04900       WRITE(4,100)BB(I,1),BB(I,2)
04910   352 CONTINUE
04920       CALL PROG3(KSUM,AA,WR,WI,Z,IV1,FV1,BB,G)
04930   900 RETURN
04940       END
04950C
04960C
04970       SUBROUTINE PROG3(N,AA,WR,WI,Z,IV1,FV1,BB,G)
04980       COMMON KRET(42)
04990       DIMENSION AA(N,N),WR(N),WI(N),Z(N,N),IV1(N),FV1(N),G(2,N),BB(N,2)
05000C
05010C      READ IN FEEDBACK GAINS OUTPUT FROM PROGRAM: MODAL
05020C
05030       CALL GETPF(5HTAPE3,5HGAINL,0,0)
05040       DO 24 I=1,N
05050       READ(3,*)(G(J,I),J=1,2)
05060    24 CONTINUE
05070       CALL MATMUL(BB,G,Z,N,2,N)
05080       CALL MATADD(AA,Z,AA,N,N)
05090C
05100C      CALCULATE CLOSED LOOP EIGENVALUES
05110C
05120       CALL RG(N,N,AA,WR,WI,1,Z,IV1,FV1,IERR)
```

```
05150+'IMAGINARY PART')
05160       DO 30 I=1,N
05170       WRITE(6,25)I,WR(I),WI(I)
05180    25 FORMAT(20X,I2,'.',2X,E15.4,15X,E15.4)
05190    30 CONTINUE
05200       RETURN
05210       END
```